Mechanics of Tillage and Traction

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Mechanics of Tillage and Traction

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LESSON 1. HISTORY OF TILLAGE

1.1. INTRODUCTION

Tillage means the preparation of the growth zone in the soil (about 10 to 90 cm of the top layer of soil) for plant development. As large areas of the surface of the earth are subject to tillage, man has tried to ease the cumbersome and time-critical work of tillage and developed machines which allow in most places of the world to perform this task with ease and efficiency.

1.2. GENERAL IMPORTANCE OF TILLAGE OPERATIONS

The task of tillage is to prepare soils for productive use. Usually tillage is limited to the arable layer of soil, which contains organic matter and where plant life actually can occur. Tillage has to be performed to clear virgin soils of plants and animals for agricultural use. Furthermore, it must be performed to bring the seedlings into the soil and procure for them a good environment for further development.

Another objective of tillage is to control weeds and animals living in the soil, such as mice or slugs. This is, compared to the use of chemical means, an energy and time consuming way to control pests. Another important point is surface leveling because most operations in mechanized agriculture depend on level surfaces. Irregularities in the soil niveau may be caused by traffic on the soil, harvesting or climatic effects. Together with this goes the need to distribute clods and porosity according to plant need. The seeds should be covered by small clods for protection while around the seeds, fine soil should prevail. Under the seeds, porosity must not be too high, while smaller and larger clods should give structure to the soil. Producing this distribution of smaller and larger clods (stratified seedbed) is one of the main objectives of primary tillage.

Producing fine soils for the environment of the seedling and the structure of the seedbed is the main objective of secondary tillage and seedbed preparation. Warming up the soil and bringing air to deeper layers stimulates life in the soil. At the same time, loosening makes it easier for plant roots to penetrate into deeper soil layers. An optimum porosity will also facilitate the infiltration of air and water for the plant roots, and the ascention of water from deeper soil layers during dry periods. Loosening the subsoil may be necessary to break up a hardpan, which can be created by trafficking and smearing the bottom of the tillage zone as it happens with plowing or which may develop naturally as in sodopol soils. Finally, it can be necessary to undertake soil improvements such as bringing down organic matter into the sterile subsoil or bringing up sand/clay subsoil into arable layers containing too much sand/clay in their texture.

1.3. APPROPRIATE TILLAGE ACCORDING TO SOIL CONDITIONS

An important characteristic of agricultural soil is its texture. It is usual to divide the smallest mineral particles forming the soil matrix into the three diameter classes of sand (particles between 2 mm and 0.05 mm), silt (particles between 0.05 mm and 0.002 mm) and clay (particles smaller than 0.002 mm) [2]. Gravel and cobbles (over 2 mm) appear in agricultural soils but are usually unwanted because they make tillage hazardous and keep little organic matter. Sand-sized and larger particles can be
fractionated by sieving. Silt or clay particles must be estimated by hydrometer or pipette analysis. Soils are classified into several types, according to the distribution of the three particle size classes.

### 1.4. SOCIO-ECONOMICAL ASPECTS OF TILLAGE

Research carried out in the United States showed that, as the amount of tillage decreases (from conventional tillage to reduced tillage or no-till), the size and the number of machines decreases and costs for machinery and labor decrease, too [3]. Major costs are affected by machinery and herbicides. For corn production, the results are summarized below:

- Ridge-till and no-till are the most profitable systems in different soil types.
- In all cases mulch-till systems (fall chisel, disk or field cultivator) are more profitable than fall plow but not as good as ridge-till and no-till.
- In poorly drained soil, ridge-till is more profitable than no-till. Slopes higher than 4% favour no-till systems.

Time requirements can be reduced by 65% and 43% when replacing conventional tillage with zero tillage and reduced tillage, respectively. At the same time, energy requirements can be reduced from more than 90 kWh/ha for conventional tillage to about 60 and 10 kW·h/ha, respectively, for reduced and zero tillage system. The adoption of soil conservation tillage systems suffers from many constraints, particularly in developing areas:

- Lack of financial means (difficult procedures to obtain credits and subsidies, high interest rates).
- Climatic conditions impose more risks with the new tillage systems.
- Low technical level of farmers due to lack of research and extension.

In developed countries, energy and labor saving and the protection of soil and water from pollution and degradation are the major aspects connected with tillage. The search for adapted systems assumes that the user’s technical level and the farmer’s financial funds are taken into account. A simulation based on data of an investigation carried out in Morocco shows that the cost of direct seeding is less than for other tillage systems if the annual planted surface is larger than 60 ha. There is an urgent need to introduce sustainable agriculture in fragile ecosystems, particularly in the developing countries where low-input farming increases the environmental degradation and perpetuates low yields and low revenue.

### 1.5. HISTORY OF TILLAGE

For thousands of years of recorded history, groups of human beings have been tilling the soil in order to increase the production of food. Early evidence indicates that simple light weight wooden ploughs, for instance, were employed extensively in the valleys of the Euphrates and Nile rivers by the year 3000 B.C. Animals in the form of oxen provided the traction necessary to pull the ploughs, preparing the soil for the seeding of barley, wheat and flax crops, (Encyclopedia Brittanica, 1979). The ploughs used during that period had no wheels or mould boards with which to invert the soil and prepare a true plough furrow. Nevertheless, they served to perform an initial breakup of the soil to a shallow depth and subsequently to cover the seeds of the crop. An example of an early Egyptian wooden plough is depicted in figure 1.
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Fig 1. Early wooden plough, Thebes, Egypt, Cirea 3000

It was more than 200 years ago that the first iron ploughs were fabricated in Northern Honan, China. At first these were small hand-drawn tools having a flat V-shaped iron piece attached to a wooden blade and handles. During the first century B.C., water buffaloes were used to pull tillage implements. Subsequently, triple-shared ploughs, plough-and-sow instruments and harrows were developed.

Ploughs have been used also in India for thousands of years. Early implements had no wheels or mould board, being composed of wedge-shaped hard wood blocks, and pulled by bullock. The soil was broken into clods but not turned over, and this primary tillage was followed by the passage of a rectangular wooden beam, also drawn by bullock, for the breaking of clods and leveling of the seedbed.

Iron plough shares appeared on Roman ploughs about 200 years ago, as well as cutting coulters knives. Still no mould board was used to turn soil cover. These ploughs were pulled by teams of oxen, up to eight per team on a heavy soil with high strength. There were reports, but no solid evidence, that ploughs equipped with wheels appeared in Northern Italy around 100 A.D.

Wheels, cutting coulters and mould boards all were included on ploughs in Europe by the year 1500 A.D., as shown in figure 2. These implements could invert the soil and make true furrows and a true seed bed. High ridges of soil were left in the fields, some of which remain in evidence today. Rather than chest yokes for animals to pull tools, padded horse collars, apparently invented in China, were attached to horses. This innovation significantly improved the animal’s ability to provide draft force. Reams of two, four, eight and more horses or oxen were often used in primary soil cultivation, depending on the strength of the soil to be tilled.

Tillage implements very similar to those now in use began to appear with the introduction of the Rotherham plough in the Netherlands, England and Scotland by the early 1700’s. The principal design features of this implement remain virtually unchanged today. Also in the 18th century, Jethro Tull promoted the use of horse-drawn cultivating hoes in wide crop row spacing. The purpose of this
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technique was to destroy weeds competing with crops and to keep the soil in between the rows in a good crumbly and friable condition for water infiltration.

More than 100 years later, Robert Ransome patented a cast-iron plough share in 1785, and a self sharpening share in 1803. Later he introduced standard parts for tillage implements which could be replaced in the field, and a double shared plough. Around the same time, the practice of “mole” ploughing began in United Kingdom to provide subsurface drainage channels in wet fields. This technique is accomplished with deep soil cutting “leg” trailing a bullet-shaped mole at the base, which leaves continuous tube-like cavities in certain plastic soils, and greatly improves the internal drainage of wetlands.

On the American organic prairie soils, problems of satisfactory tillage induced John Deere, an Illinois blacksmith, to develop a steel plough with a one piece share and mould board in the 1830’s. Animal power for traction began to wane with the introduction of the steam powered tractor in the 1860’s, beginning with the larger farm operations. The first gasoline powered tractor appeared in the United States in 1892, and many manufacturers were producing these machines in Europe and America within a few years. The steel wheels on tractors began to be replaced by rubber tires in 1932, and by 1968 it was estimated that there were over 15 million tractors in the world.
LESSON 2. SOIL-MACHINE CROP SYSTEM

2.1. INTRODUCTION

Soil dynamics is the branch of knowledge that considers the motion of soil. Soil dynamics, a phase of soil science and mechanics concerned with soils in motion, may be defined as the relation between forces applied to the soil and the resultant soil reaction. This definition does not restrict the source of the force applied to the soil; consequently, the dynamic reactions that result from the natural forces of wind, water, and other sources are also included in the definition. Reactions due to wind and water are of paramount importance in erosion and hydrology and the mechanics of these reactions are being studied. However, reactions caused by mechanical forces applied directly to the soil are considered in this course. The dynamic reactions of soil in tillage and traction affect the design and use of machines that handle soil. Because of the primary interest in the interrelations, the tool (or traction device) and the soil must be considered together.

Soil movement results from man's attempts either

i. to change prevailing soil conditions to those that are more suitable, or

ii. to use soil for support, and locomotion of vehicles.

The scope of soil dynamics thus includes soil-machine relations in both tillage and traction. It is not restricted to agricultural soils and problems since information on basic soil behavior is universally applicable.

The tremendous amount of earth construction and land forming throughout the world has made machine handling of soil increasingly important in construction, military, and mining operations. The final applications of soil dynamics knowledge may differ, but the principles are independent of application.

2.2. SOIL-MACHINE-CROP SYSTEM

The machine, which can be a tillage implement, traction device, planter, fertilizer applicator, excavator etc., is used to change the soil condition by breaking it up or compacting it. In reaction, the soil offers some resistance to the machine resulting in wear and tear, high energy demand and hence high production cost. By analyzing the soil-machine interaction, it is possible to optimize this process. On the other hand, the soil provides the environment which the crop requires for growth. In so doing, the crop also replenishes the soil with organic matter or depletes nutrients in the soil.

Analysis of the soil-crop system ensures optimization of crop yield and at the same time ensuring sustainability in terms of soil and water conservation, and energy conservation. The field of soil dynamics has been developed to identify various subsystems namely, machine, soil and crop, identify various parameters for each subsystem and to explain the interactions among the machine, soil and crop.
The **machine** is usually characterized by a number of parameters namely,

i. draught required,

ii. energy required,

iii. speed of operation,

iv. width of operation,

v. depth of operation, and

vi. rake angle.

The **soil** is characterized by,

i. cohesion,

ii. structure,

iii. angle of internal friction,

iv. cone index,

v. dry density, and

vi. soil nutrients.

The **soil-metal interface** is characterized by,

i. angle of soil-metal friction,

ii. adhesion,

iii. wear factors etc.

The **crop** is characterized by,

i. germination,

ii. weed infestation,

iii. leaf area index, and

iv. yield.

Analysis of the entire system and the subsystems provides the tool to predict the performance of the machine, crop and soil. The main aim of soil dynamics analysis is to conserve energy, soil, and water, then ensuring high yields at low costs and sustainability of production.
2.2.1. Tillage

Tillage refers to the mechanical manipulation of the soil in order to provide the conditions necessary for crop growth. In conventional crop production systems, tillage accounts for over 50% of the energy expended from land clearing to harvesting. Therefore, in trying to improve productivity of crop production, more efforts should be devoted to improving the productivity of tillage operations. Since these conditions vary with crop and soil conditions, it is important to study the different aspects of soil-machine-plant system in order to minimize the deleterious effects of the interaction among the three main components of the system (soil, machine and crop).

In a classical modern production system, a machine is used to manipulate the soil in order to provide the conditions required for the crop to grow.

The conditions required for favourable crop growth may include:

- weed control,
- providing desired soil structure,
- incorporation of residues,
- preparation of land for irrigation,
- mixing fertilizer and other soil amendments into the soil and
- destroying insects, pest etc.

Because of the various requirements, soil type and condition, there are various tillage methods covering a wide spectrum from zero tillage to conventional tillage. Whatever the type of tillage adopted, the interactions can be understood by looking at the soil – machine – crop subsystems as a complete system in itself. This is illustrated in Fig. 1.1.
Any tillage operation is basically a dynamic process. Movement of soil particles during a tillage operation is the result of the application of force by a tillage tool. The soil fails due to the action of the applied force, and soil particles move in various directions. The tool geometry, operating speed, and soil physical properties are important factors influencing the soil movement.

2.2.2. Traction

Traction is the force derived from the soil to pull a load. This force is exerted against the soil by a traction device such as a wheel, track, winch sprag, or spade. The dynamic resistance of the soil to provide traction is supplied through an interaction between the traction device and the soil. This interaction is very complex and little headway has been made in solving some of the problems that result from the interaction.

2.2.3. Tillage tools

Tillage tools are mechanical devices that are used to apply forces to the soil to cause some desired effect. The desired effects that can be produced by a tillage tool are:

i. pulverization of the soil,

ii. cutting of the soil,

iii. inversion of the soil, and

iv. movement of the soil.

Tillage tools usually produce several effects simultaneously. The ultimate aim of tillage is to manipulate a soil from a known condition into a different desired condition by mechanical means.
LESSON 3. MECHANICS OF TILLAGE TOOLS

3.1 MECHANICS OF TILLAGE TOOLS

The objective of mechanics of tillage tools is to provide a method for describing the application of forces to the soil and for describing the soil’s reaction to the forces. An accurate mechanics would provide a method by which the effects could be predicted and controlled by the design of a tillage tool or by the use of a sequence of tillage tools. Furthermore, the efficiency and economy of the tillage operation could be evaluated from the mechanics. A thorough knowledge of the basic forces and reactions is required to develop the mechanics. Such knowledge is not available at present, and soil reactions cannot even be predicted, let alone controlled. As a result, an operation is performed, the conditions are arbitrarily evaluated, and additional operations are performed in sequence until the conditions are adjudged to be adequate. Thus, today, tillage is more an art than a science.

Mechanics of tillage tools have been developed where simple tools or simple actions are involved and where forces and reactions can be described. This chapter presents several approaches that have been used to develop simple forms of soil-tillage tool mechanics. Only homogeneous soil conditions are considered. Although this approach is completely unrealistic, it does not negate the results of the studies. Complete knowledge of reactions for a homogeneous soil will provide a basis for solving problems dealing with layered soils. Interactions of importance will probably occur, but they should not present insurmountable obstacles. The approaches discussed in this chapter do not represent any final solution of the problems that are posed. The approaches, however, do represent those that have been utilized and those that may contribute to the development of a successful mechanics of tillage tools.

3.1.1. The Reaction of Soil to Tillage Tools

The reaction of soil to a tillage tool can be quantitatively described only by a mechanics. The soil can be visualized as a continuous semi-infinite mass composed of air, water, and solids arranged in some homogeneous manner. As a tool advances in the soil, the soil reacts to the tool and some action occurs. For example, the soil may move as a mass, the solids may displace the air or water, or the solids may break apart. The action of the soil’s response can be described by such qualitative terms as plowing, cultivating, and harrowing. When a quantitative description is desired, however, numbers must replace the qualitative terms.

The behavior equations were developed to quantitatively describe simple reactions of the soil to forces and also to define dynamic parameters that assess the soil. Behavior is defined here as any phenomenon that can be identified, isolated, and studied so that a behavior equation can be written to quantitatively describe the phenomenon. Thus, if an action such as plowing can be represented by the simultaneous occurrence of phenomena represented by behavior equations, a possible means is available for developing the desired quantitative description. Incorporating behavior equations into a system of equations that describes an action for a specific set of circumstances is one way to develop a mechanics. The equations of the mechanics will provide the desired quantitative description.

3.1.2. Principles for Developing Mechanics
The steps involved in the development of mechanics based on behavior equations are:

i. The action to be quantitatively described must be defined.

ii. The behavior involved in the action to be described must be recognized.

iii. In most circumstances the behavior must be incorporated into a mechanics that describes the action.

The action to be described is defined by interest from outside the action. A problem to be solved, curiosity, or merely a quest for knowledge are sources of interest. In the example of the projectile, interest determines whether the path of motion of each mathematical point of the projectile must be described or whether only the path of motion of the center of mass must be described. No set procedure can be established for defining an action because the procedure usually embodies simply defining the problem. Personal interest and the nature of the action itself will influence the definition. Until the action (defined here as the doing of something) has been at least qualitatively defined, however, the problem of quantitatively describing the action cannot be undertaken.

Because no unique structure exists, because of the mathematical complexity of the structure, and because more than one behavior is always involved, two guidelines for choosing behavior involved in an action are indicated. First, the choice of behavior must be arbitrary. In other words, for any specific action most of the behavior can be ignored. Second, the mathematical complexity suggests choosing behavior where the inputs and outputs of the behavior equation are as close as possible to the factors that will describe the action. For example, stress and strain do not lend themselves to describing the path of motion of a projectile.

When more than one behavior equation is required, a mechanics is required to combine the behavior equations. Just as no specific procedure can be given for defining an action, so no specific procedures can be given for combining behavior equations. Each situation has its own peculiarities. As suggested in the example of the projectile, including a second behavior equation may so change the result of the mechanics that little similarity remains. While the details of procedure will vary, combining behavior equations usually involves considering the equations simultaneously with boundary conditions. Simultaneous solution of the system of equations results in the desired mechanics.

3.1.3. The Complete Soil-Tillage Tool Mechanics

The reactions of soils to forces applied by tillage tools are affected by the resistance to compression, the resistance to shear, adhesion (attraction forces between the soil and other material) and frictional resistance. These are all dynamic properties in that they are made manifest only through movement of soil. Acceleration forces are not a property of soil but are also present. Nichols has shown that reactive forces of all classes of soils are dominated by the film moisture on the colloidal particles and are thus directly related to the soil moisture and colloidal content.

By following the principles, a soil tillage tool mechanics can be developed in progressive stages (fig. 1.1).

The purpose of the mechanics is to quantitatively describe the action of tillage on the soil. In the initial recognition phase, the action is observed and noted to be repetitive. The recognition phase is gradually supplanted by a qualitative phase, in which the general forces are identified and specific reactions are observed. Nearly all of the world’s literature on tillage research falls into the qualitative phase as defined here. The tool size and shape, width and depth of operation, speed of operation, and soil conditions are varied and the soil reaction is noted. The procedure involves trial-and-error methods of
solving problems. The qualitative phase has been habitually utilized for problem-solving purposes; unfortunately, relations based on trial-and-error results rarely explain the underlying basic principles. Hence, the relations generally may not be used to satisfactorily explain new and untried situations, and more trial and error studies must be made.

3.2. TILLAGE TOOL DESIGN FACTORS

The purpose of the tillage tool is to manipulate a soil as required to achieve a desired soil condition. There are three abstract design factors namely, i. initial soil condition, ii. tool shape and, iii. manner of tool movement. These three design factors control or define the soil manipulation. The results of these three input factors are evidenced by two output factors, namely, i. the final soil condition and, ii. the forces required to manipulate the soil. All five factors are of direct concern to a tillage implement designer.

Of the three input factors, the designer has complete control only on the tool shape. The user may vary the depth or speed of operation and may use the tool through a wide range of soil initial conditions. However, tool shape cannot be considered independently of the manner of movement or initial soil condition. The orientation of a tool shape with respect to the direction of travel must be defined. Different initial soil condition sometime requires different shapes. For example many different shapes of the mould board plows has been developed for different soil types and conditions.

The shape that is of concern in design is the surface over which the soil moves as a tillage tool is operated. Gill and Vanden Berg classify three shape characteristics as i. macroshape, ii. edgeshape and iii. microshape. The term macroshape designates the shape of the gross surface. The edgeshape refers to the peripheral and cross-sectional shape of the boundaries of the soil working surface. Notched and smooth disk blades have different edgeshape but the macroshape may be the same. The microshapes refer to the surface roughness.

Most tillage tool have been developed by cut-and-try methods on the basis of qualitative analysis. The manipulation-shape relation has received has greatest emphasis in the development of the mould board plow bottoms, whereas force shape relations have been of concern in subsoilers and chisel type tools. Mathematical description of the shapes are the most versatile means of representation, but tools such as mouldboard plough have complex shapes that cannot easily be representative in mathematical form. Graphic representation is often employed for plow bottoms, although mathematical analysis has been attempted and computer analysis of plow-bottom shapes is increasing.

The shape of the cutting edge can materially affect draft as well as vertical and lateral components of soil forces. For example, disk blades sharpened from the concave side penetrate more readily than blades sharpened from the convex side. Worn plowshares reduce the vertical downward force V, tend to cause soil compaction, and sometimes substantially increase draft.

The roughness of a surface over which soil slides (microshapes) influences friction forces. Surface roughness is related to the initial polish and the effect of abrasive wear, and may result locally from the rust, scratches, or small depressions. Frictional resistance can account for as much as 30% of the total draft of a mould board plow. Microshapes can also have an important effect on other aspects of soil movement, such as scouring.

3.2.1. Scouring

One of the most important aspects of sliding action of soil is scouring of a tool while it is being operated. Since the coefficient of soil-metal friction of nonadhesive soil is normally less than that of soil-
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soil friction, less force is required to move a tool through soil if sliding occurs along the metal surface. Scouring is defined as the shedding or self-cleaning of the soil through a sliding action; but scouring also requires that the soil moves fast enough so that “too much congestion" does not occur. Thus, scouring is a relative term, rather than an exact term that designates the exact point where sliding begins. In normal operation where scouring is adequate, soil flows over a tool alone: a path that is determined by the shape of the tool. In adhesive soils, when sticking occurs, a layer of soil may build up along the surface of the tool so that soil flows over a layer of soil attached to the surface of the tool. In incipient cases of sliding, the soil moves across the tool so slowly that the soil on the tool acts as a rigid body which is driven through the soil mass. Soil does not flow smoothly across the plow when this occurs.

Definition

Doner and Nichols (1934) defined the scouring at any point on a sliding surface as being approximately equal to the tangential force of the sliding added to the shear resistance of the soil minus the frictional force at the same point. They concluded from their studies that plow curvature at the wing rather than at the share would reduce soil sticking.

Factors affecting scouring

Payne and Fountaine (1954) studied the mechanics of scouring along simple surfaces and concluded that the following factors affect the scouring of a tool in soil:

1. The coefficient of soil-metal friction
2. The coefficient of soil-soil friction
3. The angle of approach of the tool
4. The soil cohesion
5. The soil adhesion

Manner of movement involves orientation of the tool, its path through the soil, and its speed along the path. For tools that travel in a straight line (i.e. not rotary or oscillating tools.), the path is usually identified by merely specifying the depth and width of cut. Orientation of a tool having a particular shape may significantly influence both the soil manipulation and the forces. Often the linkage system used to position of a tool affects both depth and orientation. When sufficient power is available, speed is the easiest design factor to vary. Increasing the speed generally increases draft but also affects soil movement and breakup.

*****😊*****
LESSON 4. ANALYSIS OF SOIL-MACHINE DYNAMICS IN TILLAGE

4.1. Analysis of Soil-Machine Dynamics in Tillage

The analysis of soil dynamics in tillage mostly involve determination of cutting forces for tillage implements as a function of soil, over burden (surcharge) tool and soil-tool factors. Once the cutting force is determined, it can be used with velocity or tool speed to obtain power requirement and specific draught using appropriate equations. A number of approaches have been used in this analysis. These include:

1. Universal Earthmoving Equation (UEE),
2. Trial Wedge Approach,
3. Stress Characteristics Approach,
4. Finite Element Approach and
5. Similitude (dimensional analysis) Technique.

4.2. APPLICATIONS OF SOIL DYNAMICS IN TILLAGE

For the analysis of soil-machine dynamics to be useful, it must be applied to solving real life problems. In other words, it must be able to contribute towards improving the processes and machinery required for providing a sustainable tillage system. In this case, such a system should modify the soil to provide optimal conditions necessary for crop growth and yield and at the same time ensuring sustainability in terms of conserving energy, soil and water and ensuring non-destruction of soil structure. Some of the applications of the practical applications include:

1. design optimization of tillage tools and traction devices,
2. development of new tillage implements and traction devices,
3. energy, soil and water conservation and,
4. providing technical and scientific basis for evolving a sound economics of tillage.

i. Optimization of tillage Tools

As already noted, there are many tillage implements from hand tools to animal drawn ploughs and different tractor mounted plough tillers, harrows, etc. Although these machines are already designed and in use, engineers continue to work on them especially with respect to modifying them to address conservation issues and other issues relating to soil structure destruction. These involve changes in the tool parameters (width, depth, sharpness, rake angle, smoothness etc) and the manner in which they engage and fail the soil depending on how the tool forces are applied to the soil. By studying and
manipulating the tool and soil parameters, it is possible to optimize the design and operation of these tools.

ii. Development of New Tillage Implements

In the development of any tillage implement or related machinery, there is need to understand soil failure pattern, soil movements, and interaction between these and the machine. These will enable the designer to determine the best way to fail the soil, the best way to make the soil move through the surface of the blade or indeed how to reduce the soil strength without necessarily inverting or pulverizing the soil. The analysis of soil dynamics also enables the designer to determine maximum tool forces, soil bearing capacity, etc which will enable him determine appropriate sizes of components of the machine.

iii. Energy Conservation in Tillage

In modern day tillage, especially with new knowledge in precision agriculture, the conventional tillage system of plough, harrow, and ridge in separate operations is no more in vogue. In the classical conventional method, the concept is to apply a force much higher than the bearing capacity or strength of the soil such that the soil fails and shatters. It is also inverted, pulverized and so on. All these consume excessive energy in terms of tractor fuel consumption. In addition, it results in frequent wearing of the tillage tool, all resulting in high cost of tillage and hence crop production. With advance in soil dynamics, it is now possible to have on-board computers that can assess the soil condition, and apply just the minimum force required at a particular place and depth, thus achieving real time process control and energy conservation.

iv. Soil and Water Conservation in Tillage

Environmental concerns have made it mandatory that tillage must ensure soil and water conservation. Thus, new and existing tillage tools must be used in such a way as to conserve the soil in terms of maintaining a stable soil structure, ensuring a good balance of soil nutrients at all times and ensuring that soil water is not allowed to evaporate excessively.

The study of soil dynamics enables the engineers and indeed the tillage practitioner to understand how to appropriately combine the tool and soil factors in such a way that the balance of the soil ecosystem is not destroyed. This has led to the emergence of a number of conservation tillage practices with their associated tools and machinery.

v. Economics of Tillage

The total understanding of soil-machine dynamics enables scientists and engineers to handle tillage as an economic venture which it is. In an attempt to develop a guide to selection of optimum tillage system for any particular soil, crop and environment, Anazodo et al, (1991) presented the optimization scheme shown in Fig. 2. To be able to apply this scheme to any situation, an in-depth understanding of soil dynamics is required.
5.1. INTRODUCTION

The term Soil has various meanings, depending upon the general field in which it is being considered. To a Pedologist, it is the substance existing on the earth's surface, which grows and develops plant life. To a Geologist, it is the material in the relative thin surface zone within which roots occur, and all the rest of the crust is grouped under the term ROCK irrespective of its hardness. To an Engineer, it is the un-aggregated or un-cemented deposits of mineral and/or organic particles or fragments covering large portion of the earth's crust.

Soil material is the product of rock. The geological process that produce soil is WEATHERING, Chemical and/or Physical.

Variation in Particle size and shape depends on

i. Weathering Process, and

ii. Transportation Process.

Variation in Soil Structure Depends on

i. Soil Minerals,

ii. Deposition Process and

iii. Transportation and Deposition.

Different types of soil produced by the different weathering & transportation process include

i. Boulders,

ii. Gravel,

iii. Cohesionless Sand (Physical),

iv. Silt,

v. Cohesive Clay (Chemical).

These soils can be Dry, Saturated - Fully or Partially. Also they have different shapes and textures.

Soil Mechanics is one of the youngest disciplines of Civil Engineering involving the study of soil, its behavior and application as an engineering material. According to Terzaghi (1948): "Soil Mechanics is the application of laws of mechanics and hydraulics to engineering problems dealing with sediments
Geotechnical Engineering is a broader term for Soil Mechanics. Geotechnical Engineering contains:

1. Soil Mechanics (Soil Properties and Behavior),
2. Soil Dynamics (Dynamic Properties of Soils),
3. Earthquake Engineering,
4. Machine Foundation,
5. Foundation Engineering (Deep & Shallow Foundation),
6. Pavement Engineering (Flexible & Rigid Pavement),
7. Rock Mechanics (Rock Stability and Tunneling), and
8. Geosynthetics (Soil Improvement)

5.1.1. Mechanics of tillage

The reactions of soils to forces applied by tillage tools are affected by the resistance to compression, the resistance to shear, adhesion (attraction forces between the soil and other material) and frictional resistance. These are all dynamic properties in that they are made manifest only through movement of soil. Acceleration forces are not a property of soil but are also present. Nichols has shown that reactive forces of all classes of soils are dominated by the film moisture on the colloidal particles and are thus directly related to the soil moisture and colloidal content.

Soils may be classified as plastic and nonplastic; the term plastic implying that the soil is moldable within a certain range of moisture contents and that it will retain its molded shape after drying. Sands or other soils containing less than 15 to 20% colloids are clay are generally considered to be nonplastic. If a plastic soil is saturated with water and then allowed to dry, it passes through the following stages, in order: sticky, plastic, friable (crumbly) and hard (cemented). The friable stage represents optimum conditions for tillage. Soil compaction by tillage implements and power units, which is a serious problem in some areas, is promoted by working the soil when too wet.

Practically all tillage tools consist of devices for applying pressure to the soil, often by means of inclined planes or wedges. As tool advances, the soil in its path is subjected to compressive stresses which, in a friable (uncemented) soil, result in a shearing action.

The shearing of soil is considerably different from the shearing of most solids, in that the reaction may extend for a considerable distance on either side of the shear plane because of internal friction and the cohesive action of moisture films.

5.1.2. Engineering Properties of Soil

Soil is composed of three phases of matter – solid, liquid (water) and gas (air) (figure 1.1). The texture (grain sizes) in a soil will determine how it will behave in the absence and presence of water.
5.2. Physical properties of soils

i. Soil texture

One aspect of the physical properties of soil, its texture, is described by the percent of particles in various size classes (Table 2.1). Particle size is the defining difference between sand, silt, and clay, but of course the size of the particle has much to do with its other properties.

<table>
<thead>
<tr>
<th>Particle class</th>
<th>Size (diameter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>Less than 0.002 mm</td>
</tr>
<tr>
<td>Silt</td>
<td>0.002 – 0.05 mm</td>
</tr>
<tr>
<td>Sand</td>
<td>0.05 – 2.0 mm</td>
</tr>
<tr>
<td>Gravel, Stone</td>
<td>larger</td>
</tr>
</tbody>
</table>

Natural soils are nearly always mixtures of sand, silt, and clay particles, as well as organic matter and stones. One convenient method of naming soils is the soil texture triangle (Figure 1.2). The sides of the triangle are axes, each representing the percentages of sand, silt, and clay that constitute the soil. Special names are assigned to various combinations as designated by the areas within the triangle.
Thus, if a soil is composed of 40% sand, 35% silt, and 25% clay, it is called a loam. This is noted by the * in the figure. Note that because of the importance of the surface area-to-volume ratio, soils with as little as 20% clay still are called clay soils. There are various laboratory methods of measuring the relative amounts of sand, silt, and clay, but a field key such as Figure 1.3 gives useful results.

ii. Porosity (n)

It is a measure of the relative amount of voids in the soil. It is the ratio of the void volume ($V_v$) to the total volume ($V_t$) of the soil sample (Figure 2.3).

$$\text{Void ratio} = \frac{V_v}{V_t} = \frac{V_s + V_w}{V_t}$$ \hspace{1cm} (2.1)

iii. Void ratio (e)

It is the ratio of the void volume ($V_v$) to the volume of the solids ($V_s$) in a soil sample.

$$\text{Void ratio} = \frac{V_v}{V_s} = \frac{V_s + V_w}{V_s}$$ \hspace{1cm} (2.2)
iv. Water content of the soil ($w$)

It is the ratio of the weight of water ($W_w$) to that of the solids ($W_s$) expressed as a percentage.

$$w, \% = \frac{W_w}{W_s} \times 100 \quad \text{(2.3)}$$

v. Degree of saturation ($S_r$)

It is the percentage of void space that is occupied by water.

$$S_r = \frac{100V_w}{V_v} \quad \text{(2.4)}$$

vi. Unit weight or density ($\gamma$)
Mechanics of Tillage and Traction

It is the ratio of the weight of the soil (W) to the volume of the soil (V). In other words, it is the weight of unit volume of soil.

\[ \gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_2} \hspace{3cm} \text{(2.5)} \]

vii. Dry density (\(\gamma_d\))

It is the ratio of the weight of solids (\(W_s\)) to the total volume of soil (V).

\[ \gamma_d = \frac{W_s}{V_s} \hspace{3cm} \text{(2.6)} \]

viii. Density of the solid particles in the soil (\(\gamma_s\))

It is the ratio of the weight of the solids (\(W_s\)) to the volume of solids (\(V_s\)).

\[ \gamma_s = \frac{W_s}{V_s} \hspace{3cm} \text{(2.7)} \]

The density of the solids in soils is found to be somewhat constant. It generally remains between 2.6 and 2.8 g cm\(^{-3}\). The average value is 2.65 g cm\(^{-3}\) for sand and silt and 2.75 g cm\(^{-3}\) for clay.

ix. Atterberg Limits

**Liquid Limit (LL):** It is the moisture content at which soil changes from a plastic solid to liquid. It is the change of consistency from plastic to liquid state.
Mechanics of Tillage and Traction

**Plastic Limit (PL):** It is the moisture content below which soil changes from plastic to crumbly solid. It is the change of consistency from brittle/crumbly to plastic state.

![Diagram of Plastic Limit](image)

**Plasticity Index (PI):** It is the difference between liquid limit (LL) and plastic limit (PL).

\[ PI = LL - PL \]  \hspace{1cm} (2.8)
LESSON 6. MECHANICAL PROPERTIES OF SOILS

6.1. INTRODUCTION

Soil is a granular medium that varies in composition from organic peat to gravel and that may contain various amounts of water. The soil physical system is continually being subjected to external forces and is, therefore, dynamic. These external forces may be environmental (climate, plants, animals, and micro-organisms) or mechanical (forces applied by man using some type of machine). The specific reaction of the soil to these forces is of interest. The forces provide the means for changing soil from one condition into another and the reaction indicates the kind and degree of change. If one is to be able either to maintain a soil condition or to change it to a more suitable condition, he must first have an understanding of soil behavior; this behavior must eventually be properly described. Soil conditions and properties, widely varying types of forces, and widely varying types of behavior must all be included in any description before the description can be satisfactory.

i. Shear strength

If a soil specimen is subjected to shear stress, the shear stress-strain diagram may look like one of the curves in Figure 2.1, depending upon the soil conditions. A highly cemented soil will result in a well defined failure point as shown by curve A. Loose soil may not show any definite failure point and the stress may increase exponentially with strain reaching some maximum value as shown by curve B. Curve C is for a soil that is well compacted but not cemented.

![Fig. 2.1. Typical shear stress diagrams for soils in three conditions](image)

The soil strength refers to the value of the shear stress on a plane within the soil sample where soil failure has taken place either by rupture or breakage. For curves A and C this point is clearly defined but for curve B soil failure is not distinct. In the case of curve C the failure is considered to have taken place by yielding or plastic flow and the asymptotic value of shear stress is taken as the shear strength for this case. The shear stress-strain curves shown in Figure 2.4 are for a given normal stress on the sample. If the normal stress is changed, the shear stress-strain diagram will change and consequently the value of the maximum shear stress will also change. An increase in the normal stress would cause
an increase in maximum shear. Therefore, the shear strength is a function of the normal stress on the failure plane.

Mohr-Coulomb failure theory states that failure in a material occurs if the shear stress on any plane equals the shear strength of the material. Furthermore, the shear strength \( s \) along any plane is a function of the normal stress \( \sigma \) on the plane, as shown below:

\[
s = f(\sigma) \quad \text{------------------} \quad (2.9)
\]

Coulomb, in 1776, conducted experiments to determine the maximum shear stress that could be applied on a plane within a sample of soil at varying levels of normal stress. He plotted the maximum shear stress values at failure against the corresponding normal stress on the failure plane and suggested the following linear relationship:

\[
\tau = c + \sigma \tan(\phi) \quad \text{------------------} \quad (2.10)
\]

where, \( \tau \) = shearing stress at soil failure

- \( c \) = cohesion
- \( \sigma \) = stress normal to plane of shear failure
- \( \phi \) = angle of internal friction

The Coulomb criterion is shown as a straight line in Figure 2.5, with an intercept on the shear stress \( (\tau) \) axis equal to \( c \) and a slope equal to \( \tan \phi \). The quantities \( c \) and \( f \) are material properties frequently called cohesion and angle of internal friction, respectively. The shear strength as defined by Equation 2.10 represents the maximum shear stress that may be sustained on any plane in a given material. The strength function is called the failure envelope since it defines the limiting stress.

Field measurements of soil shear strength.

The direct shear and triaxial shear tests are laboratory procedures to measure the shear strength. Soil samples must be taken from the field to perform these tests. The samples may get disturbed and their
shear strengths may be altered in the process. To avoid this, field methods to measure the shear strength have been developed. The first method is a round shear box which is rotated after it is inserted into the soil as shown in Figure 2.10. The box is driven into the soil until the top of the box is in contact with the soil surface. The soil is excavated carefully outside the box before applying the torque to shear the soil. The soil at the bottom of the box is sheared. The shear strength is calculated using the following equation:

\[
s = \frac{3M}{2\pi r^3} \tag{2.11}
\]

where,

- \( s \) = soil shear strength
- \( M \) = moment at failure
- \( r \) = shear box radius

Markers are placed on the soil inside the box that are visible through small holes in the top of the box. The markers are used to ensure that the soil shears uniformly.

To overcome the problem that the soil located near the outer edge of the shear box must move considerably farther than that near the center, a narrow annulus has been designed as a shear box. Shear strength for the narrow annulus shaped box is calculated from the following equation:

\[
s = \frac{3M}{2\pi(r_1^3 - r_2^3)} \tag{2.12}
\]

where, \( r_1 \) and \( r_2 \) are the inner and the outer radii of the annulus, respectively.

The field apparatus described above requires excavating the soil at the outside after inserting it into the ground. A vane type apparatus as shown in Figure 2.11 does not require excavation.
Once driven into the soil the rotation causes shear of soil along the surface of the cylinder that is generated by the vanes. This device may be used at greater depths. Measurements can be made at increasing depths without extracting the shear device so that a rather complete strength profile of natural soil conditions can be obtained. The vanes have a height-to-radius ratio of 4:1. The vane shear apparatus provides no means of varying normal load. Shear strength is computed as:

\[
S = \frac{3M}{28\pi r^2}
\]  

(2.13)

where, \( r \) is the radius of the circle inscribed by vane tips.

ii. Friction

All tillage operations involve a sliding action of soil over some surface of the tool. Friction of soil against a tool having large contact areas represents a significant component of the draft requirement. Friction is also involved when two rigid bodies of soil move with respect to each other.

Hence, there are three types of frictional parameters in problems involving soil dynamics. These are soil metal-friction (μ’), soil-soil friction (μ), and soil internal friction (tan φ). Soil internal friction (tan φ) has been discussed in reference to soil shear strength in equation 2.10. To determine soil-soil friction and soil-metal friction, we make use of Coulomb’s concept of friction coefficient,

\[
\mu \text{ or } \mu' = \frac{F}{N} = \tan \psi
\]  

(2.14)

\( \mu = \text{coefficient of friction (soil on soil)} \)

\( \mu' = \text{coefficient of friction (soil metal-friction)} \)

\( F = \text{frictional force tangent to the surface} \)

\( N = \text{normal force (perpendicular to the surface)} \)

\( \psi = \text{friction angle} \)
iii. Adhesion

Adhesion is defined as the force of attraction between two unlike bodies. In soil, adhesion is due to the film of moisture between soil particles and the surface contacting the soil. The force of adhesion is due to the surface tension of water and consequently it depends upon the value of surface tension and moisture content of the soil. However, it is virtually impossible to differentiate between friction and adhesion. Thus, an apparent coefficient of friction is often used to include effects of both friction and adhesion. Figure 2.13 shows the effect of moisture content on the apparent coefficient of friction.

It can be seen that initially at low moisture content the friction is due to pure sliding action. As the moisture content increases, friction increases due to increased adhesion. As the moisture content is increased even further the friction reduces due to the lubricating effect created by the moisture film. The following model has been proposed to include adhesion:

\[
F = aC_\alpha + N \tan \theta
\]

(2.15)

Where,

\( C_\alpha \) = adhesion and
\( a \) = surface area.

iv. Abrasion by soil:

Abrasiveness is a dynamic property of soils that has a cumulative effect rather than an immediate effect. When a large amount of soil slides over the surface of a tillage tool, abrasive wear may change the size, shape or roughness of the tool enough to make it ineffective, especially if soil pressure against the tool are high. Soil characteristics or conditions that effect abrasiveness includes hardness, shape and size of the soil particles, the firmness with which the particles are held in the soil mass and the soil moisture content. The abrasive resistance of metals is influenced by its hardness, strength and toughness.

A layer coating of a special, abrasion-resistant alloy is often applied along the cutting edges of tillage tools to reduce wear rates, especially for operation in sandy and sandy loam soils. This process is known as hard facing or hard surfacing. Hard facing materials of different compositions are available for specific combinations of abrasion and impact conditions. These materials sold under various trade names are extremely hard and some are quite brittle. They are generally nonferrous, chromium-cobalt-tungsten alloys, or high-carbon iron base alloys containing such elements as chromium, tungsten,
manganese, silicon and molybdenum. They are applied to plowshares, subsoiler points, chisel cultivators and other tillage tools by means of electric arc or an acetylene torch.

v. Compressibility

Failure of a soil by compression is generally associated with a reduction in volume. Failure in shear and failure by compression are not independent phenomena, but occur as some combined action. Failure or yielding of a soil can also be evidenced as plastic flow without the usual shattering and developing of shear failure surfaces. An example is a “flowing” of a wet clay soil around a subsoiler shank as the tool moves through the soil.

vi. Erodibility

Fine grained materials are easily transported, but clays are difficult to initially erode due to cohesion.

vii. Permeability

It is defined as the ease with which water flows through the soil. Fine grained materials are less permeable.

viii. Corrosion

It is defined as the tendency to corrode materials and structures, esp. metals, laid in the ground.
LESSON 7. ASSESSMENT OF THE DYNAMIC PROPERTIES OF SOILS

7.1. INTRODUCTION

Dynamic properties are capable of characterizing soil as it reacts to applied forces. Interrelations between applied forces, dynamic properties, and behavior are implied in figure 3.1. The figure also suggests the qualitative nature of these relations. The soil and the applied forces are both of importance so that once a specific soil is isolated and a specific system of forces applied, the resulting behavior is determined. The behavior must, be considered in a restricted form; hence, a statistical form of result is not produced. The soil and forces thus are primary and basic, and the behavior is derived from these basic factors.

Dynamic parameters provide a means for numerically representing dynamic properties. The parameter is defined as a quantity or constant whose value varies with the soil conditions or circumstances. Each equation that defines a dynamic property contains one or more mathematical parameters that are measures of the dynamic property. Thus, for a given circumstance (soil in a specific condition), the numerical value of the parameter is a constant; but, as the soil condition varies because of such things as a change in moisture content, the numerical value of the parameter may also vary. In other words, the dynamic parameters are precisely determined by the physical properties of the soil although the exact relation may not be known. A complete quantitative description of behavior of the soil to forces can be determined after one has identified, qualitatively defined, and finally quantitatively assessed dynamic parameters of soil.

7.2. MEASURING INDEPENDENT PARAMETERS

By using appropriate apparatus, specific independent dynamic parameters of shear, tension, compression, plastic flow, friction, and adhesion can be measured under highly controlled conditions. The methods that have been developed are not completely accurate or satisfactory. Undoubtedly, these measurement techniques will be modified or improved as research is continued; however, techniques for a number of measurements have been standardized to some extent and an effort should be made to use these when practicable.
7.2.1. SHEAR STRENGTH DETERMINATION

Soils consist of individual particles that can slide and roll relative to one another. Shear strength of a soil is equal to the maximum value of shear stress that can be mobilized within a soil mass without failure taking place. The shear strength of a soil is a function of the stresses applied to it as well as the manner in which these stresses are applied. A knowledge of shear strength of soils is necessary to determine the bearing capacity of foundations, the lateral pressure exerted on retaining walls, and the stability of slopes.

A. Laboratory Methods of Shear Strength Determination

Direct shear test and the triaxial test are the two most widely used methods for determining soil shear strength. The purpose of these tests is to determine the value of cohesion (c) and angle of internal friction (f) needed in Equation 2.10 to define the soil shear failure envelope.
i. Direct Shear Test

The test is carried out on a soil sample confined in a metal box of square cross-section which is split horizontally at mid-height. A small clearance is maintained between the two halves of the box. The soil is sheared along a predetermined plane by moving the top half of the box relative to the bottom half. The box is usually square in plan of size 60 mm × 60 mm. A typical shear box is shown in figure 3.2.

![Direct shear stress apparatus](image)

If the soil sample is fully or partially saturated, perforated metal plates and porous stones are placed below and above the sample to allow free drainage. If the sample is dry, solid metal plates are used. A load normal to the plane of shearing can be applied to the soil sample through the lid of the box.

Tests on sands and gravels can be performed quickly, and are usually performed dry as it is found that water does not significantly affect the drained strength. For clays, the rate of shearing must be chosen to prevent excess pore pressures building up. As a vertical normal load is applied to the sample, shear stress is gradually applied horizontally, by causing the two halves of the box to move relative to each other. The shear load is measured together with the corresponding shear displacement. The change of thickness of the sample is also measured. A number of samples of the soil are tested each under different vertical loads and the value of shear stress at failure is plotted against the normal stress for each test. Provided there is no excess pore water pressure in the soil, the total and effective stresses will be identical. From the stresses at failure, the failure envelope can be obtained.

**Advantages of direct shear test**

1. It is easy to test sands and gravels.
2. Large samples can be tested in large shear boxes, as small samples can give misleading results due to imperfections such as fractures and fissures, or may not be truly representative.
3. Samples can be sheared along predetermined planes, when the shear strength along fissures or other selected planes are needed.

**Disadvantages of direct shear test**

1. The failure plane is always horizontal in the test, and this may not be the weakest plane in the sample. Failure of the soil occurs progressively from the edges towards the centre of the sample.
2. There is no provision for measuring pore water pressure in the shear box and so it is not possible to determine effective stresses from undrained tests.
3. The shear box apparatus cannot give reliable undrained strengths because it is impossible to prevent localised drainage away from the shear plane.
ii. Triaxial Test

Triaxial test.

The triaxial test is carried out in a cell on a cylindrical soil sample having a length to diameter ratio of 2. The usual sizes are 76 mm x 38 mm and 100 mm x 50 mm. Three principal stresses are applied to the soil sample, out of which two are applied water/air pressure inside the confining cell and are equal. The third principal stress is applied by a loading ram through the top of the cell and is different to the other two principal stresses. A typical triaxial cell is shown in figure 3.3.

Consider a cylindrical soil sample subjected to a hydrostatic stress $\sigma_3$ as shown in Figure 3.4(a) and then an additional normal stress called the deviator stress ($\sigma'$) as shown in Figure 3.4(b).

The deviator stress is increased until the soil fails. Figure 3.5(a) shows a two dimensional representation of stresses on the soil specimen. The soil failure plane orientation is shown by an angle ($\theta$) from the horizontal. Figure 3.5(b) shows the shear and the normal stresses on the failure plane. Since the specimen failed the shear stress on this plane is equal to the shear strength. We now have to determine the values of the shear stress ($\tau$) and the normal stress ($\sigma$) on this plane. These stresses can be determined by means of Mohr’s circles as shown in Figure 3.6. Point A on the circle represents the failure plane. It should be noted that the angle or orientation of the failure plane is doubled in the Mohr’s diagram. The coordinates of this point are the shear and the normal stresses on the failure plane. Using this diagram the following relationships can be written for these stresses:
Figure 3.3 is a schematic diagram describing the triaxial apparatus and the application of stresses. The cylindrical soil specimen is enclosed within a thin rubber membrane and is placed inside a triaxial cell. The cell is then filled with a fluid. The specimen is subjected to a hydrostatic compressive stress ($\sigma_3$) by pressurizing the cell. This causes the soil sample to consolidate. An additional vertical stress ($\sigma'$) is applied through the piston as shown in the figure. This deviator stress is steadily increased until failure of the specimen occurs. The specimen fails under a set of principal stresses $\sigma_3 + \sigma'$ and $\sigma_3$.

Drainage of water from the specimen is measured by a burette, and valve A can be closed to prevent drainage from the specimen. Another line from the base leads to a pressure sensor to measure pore water pressure.

To obtain the failure envelope, several triaxial tests are performed on specimens of the same soil at different values of cell pressure ($\sigma_3$). A Mohr’s circle is drawn for the principal stresses at failure for each specimen. These are shown in Figure 2.10 and the line tangent to these circles constitutes the failure envelope. The stress on the failure surface is represented by the point of tangency. From the geometry of Mohr’s circle this plane makes an angle of $(\pi/2 + \phi)/2$ with the major principal stress plane.

The triaxial test may be performed as a drained (d), consolidated-undrained (c-u), or undrained test (u). In the drained test, water is allowed to seep out of the sample during the application of the hydrostatic and deviator stresses, and the pore water pressure is equal to zero. During the c-u test drainage is permitted during the application of the hydrostatic stress and the corresponding pore water pressure $u_a = 0$. When the deviator stress is applied, drainage is not permitted and the pore water pressure $u_b >$
In an undrained test, no drainage is permitted and the total pore water pressure is equal to \( u \). The effective stress \( \sigma \) for the three drainage conditions may be calculated using the following equations:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drained</td>
<td>( \bar{\sigma} = \sigma_1 \bar{\sigma}_3 = \sigma_3 )</td>
</tr>
<tr>
<td>Consolidated-Undrained</td>
<td>( \bar{\sigma}_1 = \sigma_1 - u_b ) ( \bar{\sigma}_3 = \sigma_3 - u_b )</td>
</tr>
<tr>
<td>Undrained</td>
<td>( \bar{\sigma}_1 = \sigma_1 - u ) ( \bar{\sigma}_3 = \sigma_3 - u )</td>
</tr>
</tbody>
</table>

Figure 3.7 illustrates typical Mohr’s envelopes obtained from undrained, drained, and consolidated-undrained triaxial tests; the envelopes are constructed from the principal stresses at failure. The failure envelope corresponding to the drained test, called the effective failure envelope, may be determined from Equations 3.2 and 3.3 depending on the drainage condition of the test.

Regardless of the type of the test performed, there exists an effective failure envelope unique to the soil being tested. The effective-stress failure envelope is written as:

\[
s = \bar{\sigma} + \bar{\sigma} \tan \phi
\]

\[\text{--------- (3.4)}\]

**Shear strength of cohesionless soils.**

Sand and silt are cohesionless soils. Figure 3.8 shows a typical failure envelope of a cohesionless soil.
The envelope passes through the origin. Thus, only one Mohr’s circle is needed to establish the failure envelope. The following equations are used to determine the drained (effective) failure envelope for cohesionless soils.

\[ s_{cu} = \sigma \tan \phi_{cu} \]  \hspace{1cm} \text{(3.5)}

and

\[ s = \bar{\sigma} \tan \bar{\phi} = (\sigma - u) \tan \phi \]  \hspace{1cm} \text{(3.6)}

The value of \( \bar{\phi} \) for cohesionless soils ranges from about 28 to 42°. Generally, the value of \( \bar{\phi} \) increases with increasing density. Extremely loose sands with an unstable structure may have a \( \bar{\phi} \) as low as 10°.
8.1. Field measurements of soil shear strength.

The direct shear and triaxial shear tests are laboratory procedures to measure the shear strength. Soil samples must be taken from the field to perform these tests. The samples may get disturbed and their shear strengths may be altered in the process. To avoid this, field methods to measure the shear strength have been developed. Two methods are most commonly used for measuring the soil shear strength in the field. They are,

i. Round shear box apparatus

ii. Vane type shear apparatus

i. Round shear box apparatus

The first method is a round shear box which is rotated after it is inserted into the soil as shown in Figure 4.1. The box is driven into the soil until the top of the box is in contact with the soil surface. The soil is excavated carefully outside the box before applying the torque to shear the soil. The soil at the bottom of the box is sheared.

![Fig. 4.1. Field shear apparatus](image)

The shear strength is calculated using the following equation:

\[
S = \frac{3M}{2\pi r^3}
\]

\[\text{-------------- (4.1)}\]

where,

\[s = \text{soil shear strength}\]
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M = moment at failure

r = shear box radius

Markers are placed on the soil inside the box that are visible through small holes in the top of the box. The markers are used to ensure that the soil shears uniformly.

To overcome the problem that the soil located near the outer edge of the shear box must move considerably farther than that near the center, a narrow annulus has been designed as a shear box. Shear strength for the narrow annulus shaped box is calculated from the following equation:

\[ S = \frac{3M}{2\pi(r_1^2 - r_2^2)} \]  \hspace{1cm} (4.2)

where, \( r_1 \) and \( r_2 \) are the inner and the outer radii of the annulus, respectively.

The field apparatus described above requires excavating the soil at the outside after inserting it into the ground.

ii. Vane type shear apparatus

A vane type shear apparatus is shown in figure 4.2. It does not require excavation as in the case of round shear box apparatus.

Once driven into the soil the rotation causes shear of soil along the surface of the cylinder that is generated by the vanes. This device may be used at greater depths. Measurements can be made at increasing depths without extracting the shear device so that a rather complete strength profile of natural soil conditions can be obtained. The vanes have a height-to-radius ratio of 4:1. The vane shear apparatus provides no means of varying normal load. Shear strength is computed as:

\[ S = \frac{3M}{28\pi r^3} \]  \hspace{1cm} (4.3)

where, \( r \) is the radius of the circle inscribed by vane tips.
ii. Friction

All tillage operations involve a sliding action of soil over some surface of the tool. Friction of soil against a tool having large contact areas represents a significant component of the draft requirement. Friction is also involved when two rigid bodies of soil move with respect to each other.

Hence, there are three types of frictional parameters in problems involving soil dynamics. These are soil metal-friction ($\mu'$), soil-soil friction ($\mu$), and soil internal friction ($\tan \varphi$). Soil internal friction ($\tan \varphi$) has been discussed in reference to soil shear strength in equation 2.10. To determine soil-soil friction and soil-metal friction, we make use of Coulomb’s concept of friction coefficient,

$$\mu \text{ or } \mu' = \frac{F}{N} = \tan \psi$$

$\mu$ = coefficient of friction (soil on soil)

$\mu'$ = coefficient of friction (soil metal-friction)

$F$ = frictional force tangent to the surface

$N$ = normal force (perpendicular to the surface)

$\psi$ = friction angle

An apparatus to measure soil-metal friction is shown in Figure 4.3.

Frictional force corresponding to different normal loads are measured and plotted against the normal loads. The slope of the line is the coefficient of friction. It must be pointed out that there is a difference between the soil-soil friction and the internal friction angle. In soil-soil friction phenomenon, the soil moves as a rigid body against another soil surface.

The internal friction of soil comes into play when soil fails under shear loading. Therefore, if we continue to apply shearing load in a shear test after failure, then we will measure soil-soil frictional behavior.
LESSON 9. DESIGN OF TILLAGE TOOLS

9.1. INTRODUCTION

Tillage is the manipulation of soil by mechanical forces. The purpose of tillage tool design is to create a mechanical system, that is, a tillage machine or a series of machines capable of controlling the applied forces in order to achieve a desired soil condition. As a matter of definition, a tillage tool will be considered a single soil working element whereas a tillage implement or machine will be considered a group of soil-working elements. A tillage implement or machine will include the frame, wheels, or other structural units that are needed for guidance and support. Although tillage is nearly always effected with an implement, the emphasis here will be on the design of tillage tools rather than implements.

The pressing need for design information has demanded that methods for design be developed. In fact, the need is so great that qualitative procedures have been and still are widely used. The qualitative procedures have often been based on art rather than science (121, 269). That these procedures must be changed if progress is to be made in tillage tool design, is clearly demonstrated by the history of tillage tools.

Basic tools such as the forked stick date back into antiquity; yet, they are still found in their original form in many parts of the world. Even in more advanced societies, today, the moldboard plow is designed by empirical methods. Generally, these empirical methods are trial-and-error attempts; the tool is varied in some manner and acceptable designs are identified when the resulting soil condition is adjudged to be satisfactory. Quantitative descriptions or representations of the final soil condition are seldom used and, in addition, the forces required to move the tool are frequently not quantitatively assessed. Generally, no effort is made to describe the reaction of the soil. Consequently, design today merely accepts what occurs; it does not control what occurs. Thus, even though the need for design is great, design in the true sense of the word is not accomplished and probably will not be accomplished until quantitative information is available.

To illustrate the pressing need for design information, consider the economic possibilities of the results of better design. In the United States, more than 250 billion tons of soil are estimated to be stirred or turned each year (268). To plow this soil once requires 500 million gallons of gasoline costing $105 million. If proper design could decrease the draft of the plow only 1 percent, a savings in direct operating cost of $1 million per plowing would result. If soil manipulation can be controlled by proper design so that subsequent operations may be minimized or even eliminated, additional savings would result. Control cannot be assigned realistic dollar values today because its economic effects are not known. The benefits of control in road building, land leveling, and plant growth, however, must be considered.

9.2. DESIGN FACTORS

In tillage tool design, a limited number of abstract factors become of primary importance. In order to utilize the capability of soil for some specific purpose, the soil must be manipulated (changed, moved, or formed) to a desired condition. The manipulation is accomplished with a tillage tool by moving the
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tool through the soil. To obtain different final soil conditions, only the shape of the tool and the manner of moving the tool can be varied.

The major design factors that define the soil manipulation are,

1. initial soil conditions,
2. shape of the tillage tool, and
3. manner of moving the tool
4. forces required to move the tillage tool through the soil
5. results of the manipulation i.e., the final soil conditions

The first three factors are not clearly defined in a quantitative sense although qualitatively they represent distinct and complete elements in tillage tool design. Hence, they are called abstract design factors.

Unlike the above three design factors, the forces can be quantitatively defined. These forces are not those that are applied to the soil; they are those that must be applied to the tool to move it. The tool, in turn, applies equal but opposite forces to the soil. The five design factors represent the five elements that are of direct concern and, hence, of importance to a tillage tool designer.

Relations between the various design factors provide a means for designing a tillage tool. These relations can be qualitatively determined from available knowledge of the physical action of a tool that is manipulating soil. The concept of a mathematical function is useful in representing the relations. Two real variables are mathematically defined to be functionally related within some range if a definite single value of one of the real variables corresponds with a definite single value of the other variable according to some rule. The rule that prescribes the corresponding value is the functional relation.

The concept can be extended to several variables. A functional relation exists if a definite single value for each independent variable corresponds to a definite single value for a dependent variable according to some rule. For example, if a dependent variable is a function of four independent variables, specifying the value of each of the four independent variables determines the value of the dependent variable. Consider the number of single value correspondence rules possible for the five design factors. If tool shape is physically varied but the manner of movement, and the initial soil conditions are kept constant, the forces required to move the tool and the resulting soil conditions vary as tool shape is varied. Furthermore, for each “value” of tool shape, a definite “value” of the forces and final soil condition exists. If a definite value does not exist, no unique law of nature exists. Available knowledge indicates that some kind of law does exist and tool shape does affect tool forces and the resultant soil condition. Thus, in mathematical terms, **shape, forces, and final soil condition** are functionally related.
LESSON 10. DESIGN OF TILLAGE TOOLS

10.1. Shape

The three design factors involved in the design of tillage tools are,

i. shape,

ii. manner of movement, and

iii. the initial soil condition.

Of the above three design factors, the designer has complete control only over shape. The user of a tillage tool may vary the manner of tool movement namely, the depth or speed of operation and may use the tool through a wide range of initial soil conditions. The shape of tillage tools, therefore, has received considerable emphasis since the ideal tillage tool should perform satisfactorily over wide ranges of soil conditions and depths and speeds of operation. Tool shape cannot be considered independently of its manner of movement or the initial soil condition. A description of tool shape must also be oriented with respect to the direction of travel of the tool, or its operating geometry will be ill defined. Likewise, different initial soil conditions sometimes require different shapes. This situation has resulted in the development of various configurations of moldboard plows for different soils and conditions. Since shape is the only design factor over which the designer has complete control, it must be given primary consideration in tool design.

When considering shape as a design factor, one of the first steps is to describe the shape of a tool. A description is required for the design equations as well as for construction purposes. Many tillage tools have complex shapes, and these shapes cannot easily be represented in mathematical form. Therefore, the shape description of a tool is almost exclusively restricted to the surface that comes in direct contact with the soil. Generally, no attempt is made to represent other surfaces of the tool mathematically, except when clearance is required to prevent undesirable contact with the soil.

A mathematical description of the shape of a tool is the most versatile means of representation. For some simple shapes, a plane adequately represents the surface; and the plane along with its orientation is relatively easy to describe mathematically. As more complex surfaces have evolved, however, standard mathematical equations often no longer represent the surfaces. Attempts have thus been made to find other ways to describe the complex shapes. These attempts often employed graphic representations of shapes that cannot easily be represented by mathematical equations. Some representations of tool shapes have been carried to the extreme so that the shape is described only by the pattern or mold used in its manufacture. The development of means to adequately describe the of tillage tools is still a major area of concern in the design of tillage tools.

A second step associated with shape as a design factor requires identification of the variables that represent the shape design factor. A third step is the establishment of the equations themselves. Unfortunately, these last two steps may have to be combined. To illustrate, the shape variables in the design equations should be the parameters that are identified by mathematical descriptions of shape. Probably, however, geometric parameters will not be independent of each other in the design equations. Any dependence must be represented in the functional relation and the design equations.
will not be simple. On the other hand, a transition equation might be developed in which the shape description parameters are combined to define new variables that will be independent in the design equations. With independent variables, the design equations should be simpler and easier to develop. Thus, identifying the variables and developing the equations may not be separate.

In essence they are separate, since independent variables could be identified; but the functional relations could be so complex that they could not be established. Describing the shape, identifying the shape variables in the design equations, and developing the equations themselves are the three elements involved with shape as a design factor.

10.1.1. Macroshape

The surface over which soil moves as a tillage tool is operated constitutes the shape that is of concern in design. Any finite surface has a boundary or edge whose “shape” is independent of the shape of the surface itself. A tillage tool is finite and has boundaries that are generally referred to as edges. The shape of edges in contact with soil affects the forces required for soil manipulation just as does the shape of the finite surface. Since the area of the edges of a tool is generally much smaller than the area of the surface itself, the term edgshape is used to refer to the shape of edges while the term macroshape is used to designate and emphasize the shape of the gross surface.

The moldboard plow is perhaps the most widely used tillage tool today. Its ancestor is, of course, the forked stick. While its history is long, descriptions of the macrosurface of a moldboard plow did not receive much emphasis until metals were used to construct the plow. Even then, the first descriptions of the surface were developed so that the plow could be constructed. Very little emphasis was placed on determining a relation between force and shape or between soil condition and shape that could be used to evaluate designs.

Any emphasis that was placed on design resulted in information that could be utilized in equations 132 and 133 only in a qualitative sense (160). The usefulness of descriptions of shape, however, does not hinge on their utilization in design equations, since the first requirement in shape design is a description.

One of the first methods for accurately describing the surface of a moldboard plow was developed by Thomas Jefferson in 1788 (199). It was a physical method that could be used for constructing a plow. A description was inherent in the method he proposed since it involved generating a surface with a rigid framework (fig. 145). The surface was generated by using lines e-m and o-h as directrixes and by moving a straight edge from g-e to m-h in such a manner that the straight edge remained and rotated within the vz plane. The surface generated by the straight edge as it moved was considered to be the surface that would invert the soil with the least possible resistance. Jefferson did not determine the mathematical description of the surface; he merely devised the framework and the method of its use to represent a plow surface.
A number of models of this type of framework have been used in constructing moldboard plows. Whereas Jefferson used straight lines to represent both the generator (the moving line) and the directrixes (fixed lines), others have used catenaries, arcs of circles, cycloids, and helices. Never has the choice been based on more than intuition. The method does, however, provide a means of generating a vast number of different surfaces applicable to moldboard plows.

A mathematical analysis of a surface generated by this method can be made to determine an equation that describes the surface. White (502, 503) analyzed a number of plows, including the Jeffersonian plow. He established equations for the surface in a Cartesian coordinate system oriented as shown in figure 145. The surface equations were derived from the equations of three different lines that were passed through the surface. Upon a rotation of the reference coordinate system, the equations of the Jeffersonian plow could be reduced to those of hyperbolic paraboloids. White was able to obtain equations to describe other plow surfaces that were not hyperbolic paraboloids. He was unable, however, to relate these equations to either the forces or the resultant soil condition except in a general qualitative way. Nevertheless, his work is of merit because he demonstrated that an existing plow shape could be mathematically represented.

Graphical descriptions of shape have been used by various researchers including White (502, 503), Ashby (18, 19) and Krutikov and others (231). They used graphical methods both to describe the shape and to try to establish design equations. In one commonly used method for graphically determining macroshape, an apparatus similar to that shown in figure 146 is employed. The gridded plane on the right in figure 146 is used to position a measuring rod. The tool whose shape is to be described can be oriented so that the gridded plane represents a plane perpendicular to the direction of travel of the tool. If the gridded plane is a yz plane, the x and y axes are so oriented that the xy plane is a horizontal plane and the xz plane is a vertical plane; both contain the direction of travel. To describe the shape of a properly oriented tool, the distance x that the rod extends through the yz plane is recorded at each grid location in the yz plane. A two-dimensional graph can be made by plotting the x and y values at a constant z and by connecting the plotted points with a smooth curve. The curve represents the intersection of a horizontal plane and the surface being described. By superimposing a series of such contour lines in one plane and connecting the ends of the contour lines (the edges of the surface), a two-dimensional representation of the surface is constructed.
LESSON 11. MOULD BOARD PLOW SURFACES

Representations of moldboard plow surfaces prepared by this method are shown in figure 147. Soehne (402, 403) used an optical means to expedite the preparation of descriptions by this procedure. He projected a slit of light onto moldboard plows that were painted white and recorded the reflected light trace photographically. The technique provides a rapid and accurate means of obtaining descriptions of shape (fig. 148). The slit of light could be projected either vertically or horizontally so that contours could be obtained in either direction. Accuracy was assured since both the projector and the camera were fixed and only the plow bottom moved. Soehne labeled the photograph contours alphabetically from the bottom to the top and from the front to the rear, as shown in figure 149. Contour representations of the type shown in figures 147 and 149 generally have not been mathematically described. They have been useful, however, for comparing shapes and for manufacturing purposes.
One of the first attempts to relate shape to newly created soil conditions (equation 133) was made by Ashby (19). He utilized graphical representations from which he defined parameters of the shape of plow bottoms. He attempted to correlate the shape paraobservations of plow performance. His measure of performance was primarily the covering of plant residues during actual plowing. Ashby defined a slope coefficient that was identified with the “full cut section” of a plow (fig. 150). This section is located in a vertical plane drawn perpendicular to the cutting edge of the share. The vertical plane is drawn to intersect the share at a distance from the furrow wall equal to the width of cut of the plow. The intersection of the plow surface and the vertical plane forms a curved line AB, which is used to define the slope coefficient. The curved line is shown in profile in figure 151, A along with its projected distances C, H, and X. Ashby defined the slope coefficient as follows:

\[
\text{Slope coefficient} = \frac{H}{X+2C}
\]

Where, H = height of the plow,
X = projected width of the plow,
C = distance the top of moldboard departs from the vertical line tangent to the curved profile.
When the moldboard has considerable twist, the slope coefficient has to be modified by a correction factor. Another section of the plow, identified as the twist section DC in figure 150, lies parallel to the full cut section but is two-thirds of the distance from the full cut section to the wing tip. The profile of the twist section CD, shown in figure 151, B, was used to determine the slope coefficient correction factor. The correction factor was computed from the equation

\[
\text{Correction factor} = \left( \frac{E}{V} - 0.20 \right) \frac{W - H_D + V}{V}
\]

where W, V, E, and HD are distances identified in figure 151, B. The correction factor was applied only when the ratio E/V was greater than 0.20. The slope coefficient as modified by equation 139 thus becomes

\[
\text{Slope coefficient} = \frac{H}{X^2 + C} - \left( \frac{E}{V} - 0.20 \right) \frac{W - H_D + V}{V}
\]

Ashby used equation 140 to calculate the slope coefficient of a number of moldboard plows. The plows were operated in a series of field experiments during which the covering performance of the plows was judged according to a standardized rating system. A multiple correlation was made between the covering performance and the number of plow shape factors believed to affect the covering performance. The correlation resulted in the following equation:

\[
X = 0.0355A - 0.055B - 0.644C - 0.118D - 0.0357E + 4.072
\]

Where, X = subjective rating of covering performance,

A = size of the plow,

B = horizontal clearance of the plow,

C = slope coefficient,

D = width of plow at waist,

E = relation of height to width of wing.
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As the regression coefficient in equation 141 indicates, the slope coefficient $C$ appears to be the most important factor considered. Ashby’s work illustrates one of the principles discussed in the empirical development of design equations. A parameter of the shape description is used as a variable in a design equation. Although this particular slope coefficient is probably a poor parameter and inadequately reflects the shape, it nevertheless is commendable as a first attempt. One hopes that more than covering can be included as a measure of the newly created soil condition. In addition, a quantitative description of the covering should be developed to replace the standardized rating system. Although equation 141 is inadequate and probably inaccurate in terms of design needs for today, its weakness should serve as an impetus to seek improvement.

In principle this rudimentary equation demonstrates that design equations can be developed.

Soehne (402, 403) defined a number of parameters of shape descriptions and attempted to relate them to plow performance. He used the light slit technique to represent shape along grid lines l-11 and a-m, spaced 40 millimeters apart (fig. 149). And he selected the following angles as parameters:

- $\phi_{1a}$ = share cutting angle,
- $\phi_{10a}$ to $\phi_{10j}$ = lateral directional angle of the moldboard upper,
- $\phi_{5a}$ to $\phi_{5i}$ = lateral directional angle of the horizontal shape line 5 at the intersection with vertical shape line a to i,
- $\delta_{al}$ to $\delta_{ah}$ = share intersection angle at the vertical shape line a to h,
- $\delta_{a5}$ to $\delta_{i5}$ = angle of the vertical shape line a to i along the horizontal shape line 5,
- $\delta_A$, $\delta_B$ = angle of the vertical shape line at A and B.

The performance of the plow was judged by draft resistance, and coverage and movement of the soil. He correlated the performances of 25 shapes with the list of parameters and concluded that certain parameters were important and had optimum values. Since he was considering high-speed plowing, the optimum values were for a speed of 9 kilometers per hour. The most important parameters and their optimum values are:

1. The best share intersection angle $d_{al}$ is about 15° to 17° at the share point and 8° to 10° at the share end.
2. The share cutting angle $\phi_1$ should be about 35° to 38°.
3. The lateral directional angle $\phi$ at a height of 160 to 200 millimeters (approximately elevation 5 in fig. 149, B) should be equal to the share cutting angle of the land side and should decrease toward the end of the moldboard to 23° to 27°; the horizontal shape lines must have a convex curvature.
4. A strong twist, as reflected by increasing values of $d$ in the wing area i-j-k, is required for inversion at high speeds.
5. The upper edge of the moldboard should be raised higher toward the furrow side so that soil is not sprayed over it at high speeds.
Soehne’s parameters should not be considered as the ultimate parameters for describing a moldboard plow. While his optimum values are useful, they represent an initial qualitative approach to obtain design information. Quantitative values (the parameters) describe shape, but these values have not been related either to forces or to the resulting soil condition. Soehne has demonstrated, however, that more than one parameter of shape description can be obtained.

An indirect approach toward describing the macroshape of a plow, which is closely allied with the surface shape, has been used to develop design equations. In this approach, actual path of travel of soil over the macroshape surface is determined. One technique for determining the actual path is to paint a thin coat of lacquer on the tool surface prior to operation (84, 334, 402). Scratches on the surface made by the soil trace the path of travel. Figure 152 shows that the path of travel is straighter and steeper at 7.75 miles per hour than at 1.75 miles per hour.

The path of travel is related to the acceleration that is imparted to the soil; this, in turn, affects the acceleration force that contributes to draft. After examining a number of plows, Soehne concluded that so-called high-speed plows accelerated the soil less than did so-called standard-speed plows.

Carlson (60) reached the same conclusion from a study of two plow shapes in which he used a digital computer to calculate the acceleration characteristics of each shape. Even though the mathematical representation of acceleration is simple and fully developed, acceleration has not been quantitatively incorporated into design.

Nichols and Kummer (319) developed a tracing apparatus that could provide empirical equations to describe the path of soil movement over a moldboard plow (fig. 153). The principle of the apparatus was similar to that of Jefferson (199); he used a predetermined physical path along which to establish plow shape. The apparatus of Nichols and Kummer used the plow shape as the “directrix” to determine the orientation of the generator. Recall that Jefferson prescribed the orientation of the generator which thus swept out the shape.

The tracing method was based on the observation that most plow shapes represent a section of cylindrical surface. Therefore, a selected test arc may be fitted to the surface of a plow and swept across the plow along a hypothetical path of travel. As shown in figure 153, a measurement carriage rolling on guiding rails permits the test arc to be moved along the plow. At various increments of travel, which can also be considered to be units of time, the rotation angles and can be simultaneously measured. The variations in the vertical and horizontal rotation angles were plotted as functions of the carriage movement distance. The results combined Cartesian and polar coordinates into a single graphical...
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system that could be used to compare different plow shapes. A mathematical expression for the functional relation provides a description of shape. Although mathematical characterization of the curves indicates differences between shapes, the characterization provides no rigorous information that can be utilized in plow design. In spite of this lack of design information, the device established a physical link between soil and plow because the path measured by the device and the actual paths of soil moved by the standard speed plows used at that time fortuitously coincided.

Pfost (334), using the same apparatus in a slightly different fashion, simplified the procedures. He described the path of only one point at a radial distance R (fig. 153), whereas the horizontal rotational angle was maintained perpendicular to the direction of travel. Only the values of the inversion angle were recorded for each increment of S. As shown in figure 154, the constructed and the actual paths of soil travel for two plows agreed very well for one speed of plowing. Unfortunately, the speed at which the lines agree cannot be predicted, so that the usefulness of the method is limited to a descriptive role. Thus, while a description of actual path of soil travel is useful for calculating acceleration imparted to soil during movement over the surface, the description per se is no more useful for design purposes than a geometric description.
Regardless of the means by which macroshape is described, the description must ultimately be related quantitatively to forces and final soil conditions as implied by equations 132 and 133. Until such relations are effected, quantitative design information will have to originate from trial-and-error procedures.

Gross descriptions of shape, even though nondetailed, can be useful for design. Qualitative descriptions such as can be developed by the Jeffersonian technique have contributed greatly to the development of moldboard plows. The use of one such gross description was reported by Kaburaki and Kisu (205). Modifications made in the shape of moldboard-type plows have resulted in better scouring characteristics for Japanese conditions. These developments have led to an elliptical-shaped plow (fig. 155).

The exact nature of the superior frictional relations was not determined; but the tool had a smaller width of cut, which may have produced lower normal forces. Even though the design relations have not been established, the elliptical description permits a convenient shape characterization. Since the tool is symmetrical, it could also serve as a two-way plow, as shown in figure 155.
LESSON 12. PRINCIPLES OF SOIL CUTTING

12.1. INTRODUCTION

Soil cutting may be defined as the complete severing of the soil into distinctly separate bodies, a slicing action that does not result in any other major failure such as shear. Conditions under which pure cutting may occur are determined by the soil characteristics and moisture content and, to some extent, by the degree of confinement. In many tillage operations, cutting is not a clearly defined independent action.

12.2. Analysis of soil cutting and tillage

Basically, all soil cutting, moving and tillage instruments transfer soil from its original location. Thus the mechanical failure of the soil material is involved, in the sense that the mass of soil being moved does not retain its original geometric shape. The design of effective and efficient cutting tools begins with the analysis of this soil failure, in order to predict the forces and energy required by the implements. The design process proceeds subsequently to the description of soil manipulation and structural changes which result from cutting tool action, depending upon the special applications of interest.

In situation A in figure 3.1, the lift height of the cutter is sufficient to cause shear failure surfaces that reach the surface of the soil. In situation B, however, the lift height of the cutter is not sufficient to displace the soil enough to cause major shear failure. In situation B, the soil mass separates with little evidence of any other action. Presumably, the soil was capable of absorbing the strain even though the same lift height as in situation A is used. To a certain degree, therefore, cutting as an action distinct from other failure of soil is determined by the degree of confinement in the neighborhood of the cutter. Obviously, the size of the required neighborhood for a specific cutter will be influenced by the condition of the soil. A wet plastic soil may cut in situation A of figure 3.1, whereas a dry brittle soil may create shear failure even with a low lift height as shown in situation B. Cutting as affected by degree of confinement was further demonstrated by Kostritsyn (230). He studied vertical rather than horizontal cutters. The cutters could be described as thin knives, and he observed the soil movement caused by the cutter. The results of his observations are shown in figure 3.1.
He noted that near the surface, soil would rupture or move upward (fig. 3.1-A); but at deeper depths, the movement was parallel to the direction of travel of the cutter. Figure 3.1, B shows the measured draft of the cutter versus the depth of the operation. Below a critical depth, a linear relation existed between draft and depth. The critical depth generally coincided closely with the observed depth where soil movement became horizontal. Kostritsyn reported that the critical depth was 20 to 25 centimeters for cutters approximately 3 centimeters thick. Thus, at deeper depths confinement of the soil causes pure cutting, whereas at shallower depths other types of soil failure also may occur. Kostritsyn described the soil movement near the surface as a crumbling action with the formation of crescent-shaped sliding bodies of soil. The description is similar to that used by Zelenin and Payne for vertical tools. Thus, although cutting can be clearly defined, in tillage tool operations it is not always clearly and independently involved. Obviously, a gradual transition from pure cutting to some complex action occurs as the depth of operation of a vertical cutter is decreased. A similar transition must occur when the depth of operation of a horizontal cutter is decreased or its lift height is increased at a constant depth of operation. Similarly, if soil conditions change from plastic to brittle, a transition from pure cutting to a complex action may occur. This lack of isolation of cutting has probably delayed its study, and no behavior equations have been developed to represent, cutting. As a consequence, no dynamic properties of soil associated with cutting have been identified.

Detailed studies of pure cutting have been made in the U.S.S.R., and Kostritsyn (230) continued the studies to a point where he developed a mechanics of cutting. His work followed the efforts of several earlier researchers, and he relied heavily on their findings. An interesting difference exists between the development of the mechanics of cutting and the development of the mechanics of inclined and vertical tools. In the latter mechanics, the reaction of the soil to a tool was visualized to result from simple behaviors that had been identified and were quantitatively defined. Thus, such factors as shear failure and soil-metal friction were used as the basis on which to establish a mechanics. Kostritsyn, on the other hand, directly analyzed the action and in the process developed a mechanics. He did not begin with behavior equations, but his mechanics implied the existence of a simple behavior equation. Kostritsyn’s success in developing his mechanics hinged on evaluating the implied behavior equation. While he recognized the behavior, he proceeded to evaluate it by indirect means rather than by a direct study of the behavior itself. Thus, he did not develop a simple behavior equation that, in turn, identified behavior properties of the soil. Kostritsyn did, however, manage to assess cutting of soil in empirical terms—that is, numbers that are some composite of the overall action involved in cutting rather than a clearly defined independent dynamic parameter. On this basis, he was able to construct his mechanics so that he could represent the action of cutting.

To illustrate the mechanics of cutting, Kostritsyn’s analysis is presented in detail. Kostritsyn restricted his attention to situations where only pure cutting was involved. In developing his mechanics he did not consider that any major failures other than separation were present. He considered two basic shapes of cutters (fig. 3.2).
The leading angled edge of the cutters he designated as the wedge of the cutter while the parallel edges he called the sides. He reasoned that the forces acting on a cutter could be separated into several components and these components must be in equilibrium so that

\[ P = P_1 + P_2 + P_3 \]  \hspace{1cm} (3.1)

where \( P \) = total force (draft) on the cutter,
\( P_1 \) = component of resistance resulting from the normal force on the wedge of the cutter,
\( P_2 \) = component of resistance resulting from the tangential force on the wedge of the cutter,
\( P_3 \) = component of resistance resulting from the tangential force on the side of the cutter.

The forces are shown in figure 4, where \( N \) represents normal forces and \( T \) represents tangential forces. For a cutter shaped as shown in figure 3.2-A, obviously \( P_3 \) in equation 3.1 will be zero. If the simple behavior of soil-metal friction is recognized, the tangential forces can be expressed in terms of the normal forces and friction is identified as the source of the tangential forces. Figure 3.3 shows the resistance forces resolved into horizontal components so that equation 3.1 can be written

\[
P = 2N \sin \frac{\alpha}{2} + 2N \mu' \cos \frac{\alpha}{2} + 2N_1 \mu'
\]

\hspace{1cm} (3.2)

Where,
\( a \) = wedge angle of the cutter,
\( \mu' \) = coefficient of sliding friction,
\( N \) = normal force on the wedge of cutter,
\( N_1 \) = normal force on the side of cutter.

With equation 3.2, Kostritsyn could calculate the cutting force for a particular cutter if he could specify the magnitude of the normal forces. By reasoning that these normal forces resulted from the resistance of the soil to deformation, he defined measures of the resistance so that

\[ N = K_1 F_1 \]  \hspace{1cm} (3.3)
Where,  

\[ K_1 = \text{specific resistance to deformation}, \]
\[ F_1 = \text{area of the wedge of cutter}, \]

and

\[ N_1 = K_2 F_2 \]  \hspace{1cm} (3.4)

Where,

\[ K_2 = \text{specific pressure of soil}, \]
\[ F_2 = \text{area of the side of cutter}. \]

K2 differs from K1 in that K1 goes to zero when movement of the tool is stopped. K2, on the other hand, continues to press on the sides of the cutter even though the cutter is stopped; it reflects an elastic type of restoration property in the soil. With equations 3.3 and 3.4, equation 3.2 can be rewritten to give

\[ P = 2K_1 F_1 \sin \frac{\alpha}{2} + \frac{2}{2} + 2K_2 F_2 \mu' \]  \hspace{1cm} (3.5)

Equation 3.5 provides a means for experimentally studying cutting. For a specific cutter, the values of \( F_1, F_2, \) and \( \mu' \) are known or can be determined and only \( K_1 \) and \( K_2 \) are unknown. By measuring \( P \) for a cutter shaped as shown in figure 4-A, \( K_1 \) can be evaluated from equation 3.5 since \( F_2 \) will be zero. If a similar cutter, but shaped as shown in figure 3.2-B is also used, the value determined for \( K_1 \) and equation 3.5 are sufficient to evaluate \( K_2 \). Figure 3.4 shows the difference in force on cutters of several thicknesses measured by Kostritsyn.

Having introduced the concepts of specific resistance and specific pressure (\( K_1 \) and \( K_2 \), respectively) that originate because the cutter causes the soil to deform, Kostritsyn took steps to evaluate the amount of soil deformation caused by a given cutter. He accepted as a working hypothesis the results of earlier Russian researchers who demonstrated that the path of movement of a particle of soil follows the
direction of the resultant force i.e., the path of motion and line of action of the resultant force will coincide.

Figure 3.5 shows the paths of deformation for specific circumstances. If no friction occurs between the wedge of the cutter and the soil, a particle originally situated on the center axis of the direction of travel of the cutter will follow the path a’a shown in figure 3.5. During the forward movement of the cutter, the particle initially located at a’ will be moved along line a’a to a. Where soil metal friction is involved, the resultant force is inclined to the wedge by the angle of soil-metal friction. In this case, a particle is moved along the line a"a, as shown in figure 3.5. The maximum displacement of soil thus is not equal to the width of the cutter S. Because of the interaction between the wedge angle and the soil-metal friction angle, the actual path length is something greater than S. Having established the direction of soil movement, Kostritsyn could calculate the maximum deformation for a given situation. The maximum occurs when point a in figure 100 reaches the widest part of the wedge section of the cutter. The maximum deformation is shown in figure 3.5 in the triangle bb’b” where geometry indicates that the angle b”bb’’ is \( (a/2 + d) \) so that the length b”b is given by the equation

\[
L_{\text{max}} = \frac{S}{2\cos(\frac{a}{2} + \delta)}
\]

\[\text{------------ (3.6)}\]

Where,

- \( S \) = width of cutter,
- \( d \) = angle of soil-metal friction,
- \( a \) = wedge angle of cutter,
- \( L_{\text{max}} \) = maximum deformation.

The soil deformation along the wedge will vary from zero at the tip to the maximum shown in equation 3.6, so that average soil deformation \( L_0 \) can be calculated by the relation

\[
L_0 = \frac{0 + L_{\text{max}}}{2} = \frac{S}{4\cos(\frac{a}{2} + \delta)}
\]

\[\text{------------ (3.7)}\]

Equation 3.7 applies to the deformation caused by the wedge portion of a cutter. Kostritsyn reasoned that no additional deformation occurs along the sides of the cutter, and the average deformation will be constant and numerically equal to that given for half of the maximum deformation occurring on one side of the cutter and expressed by equation 3.6.
Kostritsyn argued that a relation must exist between soil deformation, specific pressure, and specific resistance. In reality, specific pressure and specific resistance are the normal stresses acting between the soil and the cutter. The value of specific resistance is the stress required to cause a given deformation. Kostritsyn recognized that such behavior is stress-strain behavior (uniaxial compressive stress-longitudinal strain), but he also recognized that no suitable relation existed. Such a relation can be represented by a simple behavior equation that would define dynamic parameters of the soil. With the direction of strain movement specified as discussed (the behavior output of the equation) the mechanics of cutting could be developed in a manner similar to that used by Soehne, for example. Recognizing the behavior equations involved and specifying the behavior outputs establishes a method by which to locate and orient the forces involved.

Kostritsyn, however, by analyzing the cutting reaction, has worked backwards and shown the need for a specific simple behavior equation. The difference that was discussed earlier between the methods for establishing a mechanics thus should be apparent. Kostritsyn did not study a stress-strain relation directly, but rather indirectly through his mechanics. For a given cutter he could calculate \( L_0 \) for the respective \( K \) values from equations 3.6 and 3.7. He could evaluate experimentally the magnitude of the two \( K \) values from equation 85 by using two cutters with different shapes. An example of the relation between \( K_1 \) and \( K_2 \) and their respective average deformations is shown in figure 8 for one soil in one condition.

Several important conclusions by Kostritsyn are based on the general nature of the curves shown in figure 3.6. First, the two \( K \) values were not constant, so that they were not parameters assessing the soil. Since they represent stresses, constant values probably should not be expected. For the mechanics, however, a parameter must be found that is constant for a given soil condition. Second, the reversed trends for the two \( K \) values as the average deformation was increased indicate that some interaction between the soil and cutter must be occurring. If not, the trends should have been the same—they should both either increase or decrease as the average deformation increases.

Third, the low value of \( K_2 \) suggests that somehow the normal stress on the sides of the cutter is reduced during the soil reaction. To illustrate, we might reason that the maximum stress caused by the wedge of the cutter remains acting on the sides of the wedge as long as the soil is forced to remain deformed. In such circumstances, \( K_1 \) and \( K_2 \) should be related by the cosine of the angle of the wedge. As the data in figure 8 indicate, the implied angle of the wedge ranges from approximately 150° to 180°. Since these angles are completely unrealistic, a logical conclusion is that the stress on the sides of the cutter represented by \( K_2 \) is reduced by some type of relief. Based partly on the foregoing reasoning and
partly on other observations, Kostritsyn proposed that the specific resistance and the specific pressure come from elastic and plastic deformations of the soil. He thus defined

\[ K_1 = K_{el} + K_{pl} \]  \hspace{1cm} (3.8)

Where,

- \( K_{el} \) = stress from elastic deformation,
- \( K_{pl} \) = stress from plastic deformation.

He further considered that \( K_2 \) represented only the elastic deformation. These definitions imply that the soil deforms plastically and elastically as the wedge of the cutter advances. On the sides of the wedge, however, the soil has “adjusted itself” by plastic flow so that only the elastic rebound of the soil causes the stress. As figure 3.5 shows, the lines of action of the hypothesized stresses are not parallel but are related by the geometry of the cutter. Since the directions of \( K_1 \) and \( K_2 \) do not coincide, \( K_2 \) cannot be equated directly to the elastic component of stress in equation 3.8. From the geometry shown in figure 100, the magnitude of \( K_2 \) in the \( K_1 \) direction gives,

\[ K_{el} = \frac{K_2}{\cos^{\frac{1}{2}}} \]  \hspace{1cm} (3.9)

With equations 3.8 and 3.9, the elastic and plastic components of stress can be calculated from the values of \( K_1 \) and \( K_2 \). Figure 3.7 shows the respective values plotted against deformation, as determined by equation 3.7. Kostritsyn determined the relation shown in figure 3.7 for several soils in various conditions.

He concluded that the general shape of the curves was the same for all soils and then proceeded to obtain a mathematical expression for the curves. He noted that the relation between \( K_{pl} \) and \( L_0 \) was very close to that of an equilateral hyperbola so he used the equation to express the relation.
Close observation of the experimental data indicated that as \( L_0 \) approached zero, the relation in equation 3.10 did not hold. This suggested that a minimum value for \( L_0 \) existed and that it was equal to one or possibly several particle diameters. The minimum value \( L_{oo} \) represents a small distance that reflects the undeformable nature of individual soil particles.

Associated with this minimum deformation is the constant \( K_0 \) given by

\[
K_0 = \frac{1}{L_{oo}} \quad \text{------------------- (3.11)}
\]

Where,

\( L_{oo} = \) diameter of soil particles,

\( K_0 = \) maximum stress to cause deformation.

Equation 3.10 was reasonably accurate as long as \( L_0 \) was greater than \( L_{oo} \). To overcome the restriction placed on equation 3.10, Kostritsyn used an equal area technique to evaluate the shape of curves and obtained the equation

\[
K_{pl} = \frac{B}{2L_0} \left[ K_0 L_{oo} + \ln \frac{2L_0}{L_{oo}} \right] \quad \text{------------------- (3.12)}
\]

Where,

\( B = \) coefficient of plasticity.

Kostritsyn then defined

\( K_{el} = K_0 - K_{pl} \quad \text{------------------- (3.13)}
\]

and by using equation 16 and the equal area technique he obtained the equation

\[
K_{el} = \frac{A}{2L_0} \left[ K_0 (2L_0 - L_{oo}) - \ln \frac{2L_0}{L_{oo}} \right] \quad \text{------------------- (3.14)}
\]

where,

\( A = \) coefficient of elasticity.

Equations 3.12 and 3.14 take into account the restriction observed in equation 16. Coefficients \( A \) and \( B \) represent empirical constants that permit a more accurate representation of the relations between \( K_{el} \), \( K_{pl} \), and \( L_0 \). Note that in equations 3.12 and 3.14, \( A \) and \( B \) represent constants for a given soil condition, as do \( L_{oo} \) and \( K_0 \). Thus, in a sense they are composite soil parameters. They are not, however, parameters of a behavior equation but rather of a mechanics for cutting.
Kostritsyn evaluated constants A and B for several conditions. For the soil represented in figure 3.7, he assumed Loo was approximately 0.5 millimeter. The coefficient B can be evaluated only from experimental data so that the value for K_pl at some arbitrary L_o is required. Given the value for B, K_pl can be determined for other values of L_o by using equation 3.12. The theoretical points shown in figure 3.7 were calculated from equation 3.12. Table 1 gives measured and calculated values for K_pl and K_el for a different soil condition. Kostritsyn stated that the large percentage of error at 1-millimeter deformation was due to the inaccuracy of experimental apparatus for such small forces and deformations. As figure 3.7 and table 3.1 show, however, the experimental and calculated values agree remarkably well. Although Kostritsyn did not show data to compare the actual draft of the cutters, agreement of the soil stresses implies that the calculated and experimental drafts would have agreed just as well. Thus, the mechanics of cutting as developed by Kostritsyn is reasonably accurate and complete.

### Table 3.1. Experimental and computed values of the plastic and elastic stresses to deformation during cutting

<table>
<thead>
<tr>
<th>Mean soil deformation (mm)</th>
<th>K_pl</th>
<th>K_el</th>
<th>Difference (Kg/cm²)</th>
<th>K_pl</th>
<th>K_el</th>
<th>Difference (Kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.3</td>
<td>5.60</td>
<td>-2.5</td>
<td>0.212</td>
<td>0.330</td>
<td>+35</td>
</tr>
<tr>
<td>3</td>
<td>5.4</td>
<td>7.00</td>
<td>+3.5</td>
<td>0.390</td>
<td>0.405</td>
<td>+4</td>
</tr>
<tr>
<td>5</td>
<td>4.1</td>
<td>4.05</td>
<td>+1.0</td>
<td>0.440</td>
<td>0.440</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>2.50</td>
<td>0</td>
<td>0.465</td>
<td>0.465</td>
<td>0</td>
</tr>
</tbody>
</table>

**SOURCE**: Kostritsyn (230).

In order to determine parameters A and B, the forces on an actual cutter must be measured experimentally. Therefore, the mechanics of cutting may seem to be a hoax. Such is not the case, however, if the means required to assess the magnitudes of A and B are recognized. As has already been implied, the parameters are defined by the mechanics rather than by a behavior equation. Kostritsyn argued that a stress-strain behavior equation must exist, but it is unknown. He proceeded to establish a relation indirectly through his mechanics where cutting is involved rather than a direct relation where the stress-strain behavior was isolated. His mechanics of cutting is thus the mathematical model that represents the situation under observation. The mathematical model defines parameters A and B; hence, they are parameters of the mechanics, not of stress-strain behavior.

The mechanics is the model and hence an actual cutter is required. Once A and B have been specified, however, they can be used and applied to that soil condition just as independent parameters such as cohesion can be applied. The mechanics of cutting thus differs from the earlier mechanics only in the
method required to assess the respective soil parameters. While the mechanics seems to be very accurate as presented by Kostritsyn, caution should be used in its application. Stress-strain behavior is probably inaccurately represented because of the indirect manner in which it was studied. No distribution of stress is admitted to exist on the cutter when a distribution would seem logical. Whether the average stress and deformation accurately represent the implied behavior has not been determined.

Kostritsyn apparently confined his experiments to wet dense soils. Experience tells us that such soils tend to be plastic and therefore probably only pure cutting was involved in the experiments. In many soil conditions plastic behavior may be less evident and the relation expressed in equation 90 may be drastically different. If a difference occurs, equations 92 and 94 may no longer accurately represent the situation. Even further, this possibility raises the question as to whether pure cutting will be present. As soil conditions change so that plastic behavior is less dominant, other types of failure may also be present and increase in importance. Kostritsyn’s mechanics applies only to pure cutting and would not represent such situations. Considerations of this kind limit the situations where the mechanics can be applied.

A final consideration of the forces on a cutter suggest a possible component of resistance along the leading edge of the cutter. Such a force is present if large hard particles are visualized as being cut by an infinitely thin cutter, as shown in figure 3.8(A).

In order for the cutter to move from A to B along the projected path (fig. 3.8(A)), several of the large particles would have to be sliced and separated. Contrast this action to the actions mechanics for cutting, where particles presumably implied in Kostritsyn’s smaller than the cutter are merely displaced but not sliced. A possible fourth term could thus be included in equation 81. This term might be extremely useful when evaluating the cutting resistance of materials in the soil, such as roots. Figure 3.8 also shows the effect a boundary condition might have on cutting. If a pile of coal were being shoveled from the top (a situation similar to fig. 3.8(A)) rather than from a smooth floor (a situation similar to fig. 3.8 (B)), the force to push the shovel into the pile would obviously be different. In the latter case, the boundary condition becomes orderly so that cutting is not required. Shoveling from the top requires deforming the aggregate of coal in an action similar to that represented by Kostritsyn’s mechanics. The individual aggregates are displaced but not cut. Presumably an action could occur wherein the coal aggregates themselves would be severed so that an additional force would be required. Cutting per se is thus simply envisioned and easily defined, but its involvement with other actions compound and confuse the practical application of cutting. Getzlaff (142) has measured the effect of stones on the draft of plows and it appears that cutting or displacing rigid bodies can require considerable force.
Fig 3.-A, Soil movement caused by a thin vertical cutter; B, relation of cutting force to depth of operation for a vertical cutter. (Kostritsyn (230).)
LESSON 13. DESIGN EQUATION

13.1. INTRODUCTION

When designing a tillage tool for the purpose of establishing a new soil condition, our interest is no longer concentrated on the dynamic progress of the reaction of soil per se but rather on a soil tillage tool system and on the results of the reaction—the final soil condition. In design, an accurate description of how soil reacts is not essential. But the results of the reaction and what can be done to control the reaction are essential. Therefore, for design, our scope of interest must be concentrated on a quantitative description of the final soil condition and on how the manipulation can be controlled.

13.2. RELATION BETWEEN SOIL AND TOOL FACTOR IN DESIGN

Consider the situation where the tool shape and manner of movement are kept constant but soil conditions are physically varied. Available knowledge indicates that for each initial soil condition (a single “value”), definite tool forces are required and a definite final soil condition results. A functional relation between initial soil condition, tool forces, and resultant soil condition represents the situation. By similar reasoning, the manner of tool movement, tool forces, and resultant soil condition are also functionally related. Consider the possibility of physically varying tool forces for constant tool shape, manner of movement, and initial soil condition. Available knowledge indicates that in a constant initial soil condition, the forces cannot be varied unless tool shape or manner of movement is changed. If a tool is operated in a soil whose condition is constant and the tool forces are not sufficiently large, the tool cannot be moved. If the forces are too large, the tool will be accelerated or its path of movement changed. Tool shape, manner of movement, and the initial soil condition, therefore, completely determine the magnitude of the forces required to move the tool. In a similar manner, tool shape, manner of movement, and the initial soil condition completely determine the resultant soil condition. Mathematically, tool shape, manner of movement, and the initial soil condition are independent variables. The tool forces and resultant soil condition are each dependent variables, and they are mathematical functions of the same independent variables.

The implied relation between design factors is schematically represented in figure 2.1.
Mechanics of Tillage and Traction

The generalized tillage relation can be mathematically represented by the two equations

\[
F = f(T_s, T_m, S_i) \tag{1}
\]

\[
S_f = g(T_s, T_m, S_i) \tag{2}
\]

Where,

\( F \) = forces on the tool to cause movement,

\( T_s \) = tool shape,

\( T_m \) = manner of tool movement,

\( S_i \) = initial soil condition,

\( f \) = functional relation between \( F, T_s, T_m, S_i \)

\( S_f \) = final soil condition,

\( g \) = functional relation between \( S_f, T_s, T_m, S_i \).

Equation (1) is force tillage equation and equation (2) is soil condition tillage equation. The two equations represent the most general situation because, as written, the functional relations \( f \) and \( g \) are completely arbitrary. Furthermore, the two functions may or may not be different. The independence of the functional relations is of interest because of a possible higher order functional relation between \( F \) and \( S_f \). If \( F \) and \( S_f \) are functionally related, equations 1 and 2 can be combined so that the relation between the design variables is represented by only one equation. Available knowledge does not conclusively indicate whether \( F \) and \( S_f \) should be related. The possibility thus exists that two separate equations inaccurately represent the generalized relations between the design factors.

Available mathematical knowledge helps to resolve the situation. Available mathematical theorems prove that \( F \) and \( S_f \) will be related only if the nature of \( f \) and \( g \) is such that their Jacobian is zero. Furthermore, the mathematics provides a means for determining the higher order relation if it exists. Thus, no mathematical restrictions are imposed on the possible functional relations if two equations are used. Two equations actually simplify the situation since each can be studied independently, although physically both equations operate simultaneously. From Equations 1 and 2, it is found that \( F \) and \( S_f \) are both dependent variables of the same independent variables. The general relation between the five design factors is, therefore, accurately represented by equations 1 and 2.

When designing a tillage tool for the purpose of establishing a new soil condition, our interest is no longer concentrated on the dynamic progress of the reaction of soil per se but rather on a soil tillage tool system and on the results of the reaction—the final soil condition. In design, an accurate description of how soil reacts is not essential. But the results of the reaction and what can be done to control the reaction are essential. Therefore, for design, our scope of interest must be concentrated on a quantitative description of the final soil condition and on how the manipulation can be controlled. The design factors and their relations indicated by equations 1 and 2 represent the desired quantitative descriptions for a scope of interest concerned with design.

The circle (fig. 2.1) hypothetically illustrates a change in our scope of interest. Inside the circle, the soil may be visualized as being manipulated. Forces cause the manipulation, so our scope of interest centers on describing the reaction of soil to forces. Outside the circle, the results of the final soil
condition and the control of the manipulation are of primary concern. The design scope of interest is thus represented by quantities operating outside the circle.

The procedural framework for designing a tillage tool is contained in equations 1 and 2. Knowledge of the functions represented in the equations would permit a designer to use equation 2 to determine the resulting soil condition and equation 1 to determine the forces required to move the tillage tool. By simultaneously considering both equations, the possible tool shapes and movements could be optimized to effect the desired manipulation with minimum force. Since the functions are not yet known, the equations cannot be used directly for design. Even in their generalized functional form, however, they inherently establish guidelines for empirical design procedures.

The total differential of equation of 1 is

\[ dF = \frac{\partial f}{\partial T_s} dT_s + \frac{\partial f}{\partial T_m} dT_m + \frac{\partial f}{\partial S_i} dS_i \]  

and similarly the total differential of equation 2 is

\[ dS_f = \frac{\partial g}{\partial T_s} dT_s + \frac{\partial g}{\partial T_m} dT_m + \frac{\partial g}{\partial T_i} dS_i \]  

Equations 3 and 4 give the reasoning behind qualitative design procedures. For example, the shape of a tillage tool can be varied and the tool in each of its various shapes can be operated in a soil of uniform condition. If the movement of the tool is not changed (depth, width, speed, etc.), any change in the forces required to move the tool or any change in the results of the manipulation must come from the change in its shape. The conclusion is valid because equations 3 and 4 become, respectively,

\[ dF = \left( \frac{\partial f}{\partial T_s} \right)_{T_m, S_i} dT_s \]  

\[ dS_f = \left( \frac{\partial g}{\partial T_s} \right)_{T_m, S_i} dT_s \]

since \(dT_m\) and \(dS_i\) are both zero for the conditions of the observations. If one particular shape produces a soil manipulation that is judged to be superior, a description of that shape provides useful design information. Only the shape would need to be quantitatively described; the forces and the resulting soil change would not. The tool with the selected shape could be operated in various soil conditions to verify its action. If the required forces and the resultant soil manipulation were judged satisfactory, it could be concluded that the selected shape is an acceptable design for these soil conditions. Describing the soil conditions even in qualitative terms provides additional design information that can be associated with the description of the shape. Such procedures have led to the development of sod bottoms, general purpose bottoms, and slat bottoms for moldboard plows. The procedures were qualitative because numerical descriptions of all of the design factors were not necessarily used and no attempt was made to relate the design factors. Quantitative design procedures involve numerically relating the design factors to each other. If \(F\) and \(T_s\) are measured (numerically assessed) in circumstances that are accurately represented by equation 5, a unique relation between \(F\) and \(T_s\) must exist. The relation can be represented graphically by plotting the variables against each other. The plot results in a curve that represents the relation, and the equation for the curve is the solution of the differential equation in equation 5. The relation between \(S_f\) and \(T_s\) could be developed in a similar manner. Repeating the measurements in different soil conditions results in the development of a family
of curves with each curve representing a constant soil condition. If the soil conditions are quantitatively described, relations between $F$ and $S_i$ and $S_f$ and $S_i$ for constant tool shape can be obtained. The equations that describe the relations provide the solutions to equations 3 and 4 when only the soil is varied. In a similar manner, tool movement can be varied when tool shape and initial soil conditions are constant to again provide equations to describe the indicated relations. Conceivably, all of these equations could be simultaneously considered or combined so that ultimately equations 1 and 2 could be developed. Quantitative design procedures, therefore, can lead to the development of the functions $f$ and $g$ in the tillage equations.

A certain group of tillage tools can be described geometrically by one equation or more. For example, all disks made from sections of spheres can be described by the equation for a sphere. The radius of the sphere and the limits that describe the section of the sphere from which the disk is made (diameter of the disk) become parameters of the geometrical description equations. Specifying a particular disk fixes the parameters of geometrical description equations just, as specifying a particular soil condition fixes the parameters of dynamic property equations. The system of equations forms a mechanics, and the solution of the system of equations can be obtained in terms of the parameters of both the soil (dynamic parameters) and the tool (geometrical parameters). The solution will be a tillage equation. From such a generalized solution, the effects of varying either the soil conditions for a specific tool or the tool for a fixed soil condition could be determined.

Another possible application of a general solution would be to consider only one specific tool. The path of motion could be expressed by equations, then mathematically varied, and the results determined. An elementary example of this approach would be in varying width and depth of operation. A generalized solution of the system of equations in terms of the parameters of soil (dynamic parameters) and path of motion parameters thus provides another tillage equation. In this instance the effects of varying either the soil path in fixed soil conditions or the soil conditions for a fixed soil path could be determined. The general procedure described here provides a means for determining tillage equations to form a soil tillage tool mechanics.

Since many design equations must exist, a technique for representing a design factor may aid in developing tillage equations. Recall that behavior equations were said to contain parameters, and these parameters assessed the contribution of the material to the behavior. In a similar manner, parameters of tillage equations can be used to represent one of the independent design factors.

To illustrate the technique, consider a situation where equation 5 is applicable. Assume that a relation between $F$ and $T_i$ is experimentally obtained and is graphically represented. The resulting curve represents the relation between $F$ and $T_i$ at a constant soil condition and a constant manner of movement. If the measurements are repeated in a different soil condition the resulting curve probably will be different. By repeating the measurements in several soil conditions, a series of curves can be developed. The difference between curves reflects the difference between initial soil conditions. Assume that the curves are all similar and that an equation can be developed that represents the curves. The equation that describes all of the curves can be said to be a general equation. To be a general equation, rather than a specific equation, it must contain parameters. These parameters will numerically assess the initial soil condition. For example, if the relation between $F$ and $T_i$ is linear, the intercept and slope are parameters of the equation and they would numerically assess $S_i$ in the tillage equation. Each different soil condition will have a different slope and intercept. If the manner of tool movement were changed, rather than initial soil conditions, a different family of curves would result. Developing a general equation to represent the curves again will define parameters. In this case they will assess the manner of movement rather than the initial soil condition. In short, the technique
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provides a means to numerically define and assess one of the independent variables in the tillage equation.

The technique has one serious limitation. Just as behavior equations define behavior parameters, so tillage equations define design parameters. Consequently, the design parameters are defined by the soil-tillage tool system. In order to assess the parameters, a particular tool must be physically operated in a manner that simulates the system being represented. Once the parameters have been assessed, all similar tools can be described by the equations. The cutting parameters were defined by the mechanics of cutting rather than by a behavior equation. The limitation could possibly be minimized if the design parameters could be related to some other defined factor in the system. For example, if the design parameters assess the soil, they must be determined by the material and state properties of the soil just as dynamic behavior parameters must be determined by these same soil properties. Establishing the relation would overcome the limitation of the technique.

The need for design information and the complexity of the relations involved between the design factors clearly indicate that both the empirical approach and the derived approach should be simultaneously followed. Each approach requires certain facilities and interests. The empirical approach requires facilities for keeping soil condition constant and for producing various shapes of tools and equipment for controlling the manner of movement. As equations 3 and 4 indicate, control must be sufficient so that any change in F or S can be attributed to the correct design factor. In the derived approach, soil behavior can often be studied with small samples of soil and rather simple apparatus. A mechanics based on behavior equations can be developed with only a pencil and paper. Facilities and personal interest, therefore, should partly determine the approach to follow. In the empirical approach, however, emphasis must be placed on establishing quantitative relations. Only when design information must be immediately available should qualitative empirical procedures be followed. Qualitative procedures can never lead to the information needed for design where control of soil manipulation is possible. Finally, one should recognize that a complete understanding of the general behavior of soil reacting to a tillage tool can be obtained only from knowledge based on scientific principles. Such principles can never be deduced from an empirical description of general behavior. Therefore, the derived approach will ultimately have to be fully developed before a complete understanding of the soil-tillage tool system can be attained.

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MODULE 5. Application of dimensional analysis in soil dynamics performance of tillage tools

LESSON 14. Dimensional Analysis

14.1. Introduction

Dimensional Analysis is a mathematical technique which makes use of the study of dimensions is an aid to the solutions of several engineering problems. Each physical phenomenon can be expressed by an equation composed of variables (or physical quantities) which may be dimensional and non-dimensional quantities. Dimensional analysis helps in determining systematic arrangements of the variables in the physical relationship, combining dimensional variables to form non-dimensional parameters. Some of the uses of dimensional analysis are as narrated below:

i. Testing of dimensional homogeneity of any equation of fluid motion.

ii. Deriving equations expressed in terms of non-dimensional parameters to show the relative significance of each parameter.

iii. Planning model tests and presenting experimental results in a systematic manner; thus making it possible to analyse the complex fluid flow phenomena.

In order to study the application of dimensional analysis to a practical problem, preliminary discussion about the dimensions is necessary.

14.2. Dimensions

The various physical quantities used by the engineers and the scientists, to describe a given phenomenon can be described by a set of quantities which are in a sense independent of each other. These quantities are known as fundamental or primary quantities. The primary quantities are mass, length, time and temperature, designated by the letters $M$, $L$, $T$, and $\theta$ respectively. All other quantities such as area, volume, velocity, acceleration, force, energy, power etc., are termed as derived quantities or secondary quantities, because they can be expressed in terms of the primary quantities. The expression for a derived quantity in terms of the primary quantities is called the dimensions of the physical quantity. For example the dimension of force can be expressed as

$$[\text{Force}] = [\text{Mass} \times \text{Acceleration}]$$

Since, $[\text{Acceleration}] = [L \over {T^2}]$

$[\text{Force}] = [ML] \over {T^2} = [MLT^{-2}]$

The rectangular bracket signifies that the dimensions of the quantity are being considered.

14.3. Dimensional Homogeneity

An equation will be said to be dimensionally homogeneous if the form of the equation does not depend on the fundamental units of measurement. In other words, an equation is said to be dimensionally
homogenous, if the dimensions of the terms on its left hand side are same as dimensions of the terms on its right hand side. For example, the equation for the period of oscillation of a simple pendulum \[ T = \sqrt{\frac{L}{g}} \] is valid whether length is measured in feet, meters, or miles, and whether time is measured in minutes, days, or seconds. Therefore, by definition, the equation is dimensionally homogeneous. If the value \( g = 32.2 \text{ ft/sec}^2 \) is substituted in the equation, there results \( T = 1.11 \sqrt{L} \). This equation is correct for pendulums on the earth, but it is no longer homogenous, since the factor 1.11 applies only if length is measured in feet and time is measured in seconds. It might be argued that the factor 1.11 itself has the dimension \( [L^{-1/2}T] \). However, dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogenous.

It can be deduced from the above definition of dimensional homogeneity that an equation of the form \( x = a + b + c + \ldots \) is dimensionally homogeneous if, and only if, the variables \( x, a, b, c, \ldots \) all have the same dimensions. If a derived equation contains a sum or a difference of two terms that have different dimensions, a mistake has been made. This principle may be applied to differential equations and integral equations, as well as to algebraic equations. It should not be assumed, however, that an empirical equation is necessarily dimensionally homogeneous.

Every measurement has two characteristics: quantitative and qualitative. Dimensional analysis deals with the qualitative aspects of measurements. Qualitative aspects of measurements are expressed in terms of either primary or secondary units. The primary units are internationally accepted reference quantities in terms of which other quantities are specified (Skoglund, 1967). For example kg, m, s, and K are respectively the primary units of mass, length, time, and temperature in SI units. The derived or secondary units are expressed in terms of primary units based on mathematical relationships (Murphy, 1950). Thus speed, which is distance/time has the unit m/s.

Fundamental or basic quantities are a set of quantities that are chosen to represent other quantities based on convenience. Fundamental quantities may not be the same as primary quantities. Often force is chosen as a fundamental quantity in engineering, although it is not a primary quantity. The dimensions of these basic quantities are used in obtaining the dimensions of other quantities. Thus, if force (F), length (L), time (T), and temperature (θ) are used as fundamental quantities, then mass will have the following dimensions:

From Newton's second law:

\[ \text{force} = \text{mass} \times \text{acceleration} \quad \text{(1.1)} \]

\[ \text{Acceleration} = \frac{\text{d}^2x}{\text{d}t^2} \quad \text{(1.2)} \]

where \( x \) is displacement, and \( t \) is time. From equations 1.1 and 1.2, mass is given by:

\[
\begin{align*}
\text{Mass} &= \frac{\text{Force}}{\frac{\text{d}^2x}{\text{d}t^2}} \\
&= \frac{F}{L/T^2} = FL^{-1}T^2
\end{align*}
\]

or, in terms of dimensions:

\[
\text{Mass} = \frac{F}{L/T^2} = FL^{-1}T^2 \quad \text{(1.4)}
\]
Here, force, length, time, and temperature are fundamental quantities. The power of dimensional analysis resides in its ability to classify equations, convert equations from one system of units to another, develop prediction equations, reduce the number of variables to be investigated in an experiment, and provide the basis for the theory of similitude (Murphy, 1950). Soil dynamics as a discipline has extensively benefited from these powerful features of dimensional analysis. This technique has been widely used during the 1960s and 1970s to develop prediction equations, reduce the number of variables to be investigated, and conduct model studies.

**Buckingham pi theorem:**

**Reduction in the number of variables**

The Buckingham Pi theorem states that the number of Pi terms ($s$) required to express a relationship between variables is equal to the number of variables involved in the process ($n$) minus the number of dimensions ($b$) required to express those variables (Murphy, 1950; Palacios, 1964), i.e.:

$$s = n - b$$  \hspace{1cm} (17)

Thus, in our example of a vertical wide blade operating in a cohesionless soil, we have five variables (cf. eq. 10) and two dimensions, resulting in three Pi terms. This theorem is particularly helpful in designing experiments, since it allows us to reduce the number of variables to be investigated, thus reducing the time required, complexity, and cost of conducting the experiments.

**Form of Prediction Equations**

The next task in developing the prediction equation is to determine the form of the prediction equation, e.g., whether equation 16 is the product or the sum of two component functions, $f_1(d/w)$ and $f_2(\varphi)$:

$$\left[ \frac{D}{\gamma w^3} \right] = C f_1 \left( \frac{d}{w} \right) f_2 \left( \varphi \right) \hspace{1cm} (18)$$

To address this issue, let us consider a general process that involves three Pi terms ($\pi_1$, $\pi_2$, $\pi_3$), i.e., $\pi_1 = \Phi(\pi_2, \pi_3)$.

The minimum number of tests necessary to determine the form of the prediction equation and verify that the form is Correct, is $(2m - 3)$ for a process that involves $m$ Pi terms (Murphy, 1950). Of these $(2m - 3)$ tests, $(m - 1)$ tests are necessary to determine the component equations, and the other $(m - 2)$ tests are necessary to verify the form of the general response function. For the case when there are only three Pi terms, we need three tests to determine the form of the prediction equation. One of these tests can be conducted while keeping $\pi_3$ constant at $\bar{\pi}_3$ (the bar is used to denote constant values). This test provides the component equation: $f_1(\pi_2)$, i.e.,

$$\Phi(\pi_2, \bar{\pi}_3) = \pi_1 \bar{\pi}_3.$$  \hspace{1cm} (19)

Similarly, component equation $f_2(\pi_3)$, i.e., $\Phi(\bar{\pi}_2, \pi_3) = \pi_1 \bar{\pi}_2$, is obtained by conducting a second set of experiments while holding $\pi_2$ constant at $\bar{\pi}_2$. Referring to figure 1, if the two component equations multiply to produce the prediction equation, then we have
where constant $C$ can be shown to be shown to be Therefore, the form of the prediction equation is:

\[
\pi_1 = \Phi(\pi_2, \pi_3) = \frac{\Phi(\pi_2, \pi_3)}{\Phi(\pi_2, \pi_3)}
\]

To verify if the form of this equation is correct, a third test should be conducted by holding either $\pi_2$ or $\pi_3$ at some other level. Let us say that $\pi_3$ is held at $\bar{\pi}_3$. Then the requirement for the prediction equation to be a product of two component equations is (Murphy, 1950):

\[
\Phi(\pi_2, \pi_3) = \frac{\Phi(\pi_2, \bar{\pi}_3)}{\Phi(\pi_2, \pi_3)}
\]

Figure 1. Graph of a prediction equation containing three $\pi$ terms derived as a product of component equations.

A similar technique can be applied if the component equations are added to produce the prediction equation. Most soil dynamics problems involve more than three Pi terms. What happens if there are $m$ Pi terms? Boyd (1966) outlined a procedure for determining complicated prediction equations. For example, if $m = 5$, then a total of seven ($2m - 3$) tests are necessary to develop the prediction equation and verify it. Of these seven tests, four tests (i.e., $m - 1$) are necessary to develop component equations. The other three tests (i.e., $m - 2$) are used for verifying the form of the prediction equation. These three equations lead to three conditions rather than just one, as shown in equation 21 for a three-parameter case. Because of experimental error, these conditions are seldom exactly equal and a better approach is necessary to validate the form of the prediction. Shafii et al. (1996) proposed an alternate technique that can verify the form of the prediction equation using a rigorous statistical approach. Their approach is summarized below:

1. Conduct $(m - 1)$ sets of tests holding all but one Pi term ($\pi_j$) constant to determine the component equation $f(\pi_j)$ as:

\[
f(\pi_j) = \Phi(\pi_2, \pi_3, ..., \pi_j, ..., \pi_m), \text{ for } j = 2, 3, 4, ..., m
\]

2. If component equations multiply to yield the prediction equation, then it is expected to be of the form:
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\[ \pi_1 = C \Phi(\pi_2, \pi_3, ..., \pi_j, ..., \pi_m) \Phi(\pi_2, \pi_3, ..., \pi_j, ..., \pi_m) \Phi(\pi_2, \pi_3, ..., \pi_j, ..., \pi_m) \Phi(\pi_2, \pi_3, ..., \pi_j, ..., \pi_m) \] (23)

3. Estimate constant \( C \) from experimental data as:

\[ C = \frac{1}{\{ \Phi(\pi_2, \pi_3, ..., \pi_j, ..., \pi_m)^{m-2} \}} \] (24)

4. Obtain \( n \) additional data points corresponding to the values of the independent \( P_i \) terms (i.e., \( \pi_j, j = 2, 3, ..., m \)) such that the values of \( \pi_j \) fall within the range of interest for each of the \( P_i \) terms [i.e., \((\pi_j)\text{min} < \pi_j < (\pi_j)\text{max}\)]. Let these data be represented as:

\( \{(\pi_{1k}, \pi_{2k}, \pi_{3k}, ..., \pi_{jk}, ..., \pi_{mk}), k = 1, 2, 3, ..., n\} \) (25)

5. Predict \( \pi_1 \) corresponding to \( \{(\pi_{2k}, \pi_{3k}, ..., \pi_{jk}, ..., \pi_{mk}), k = 1, 2, 3, ..., n\} \) using equations 23 and 24. Let these \( n \) estimated data points be represented by \( \{(p_1 \pi_{1k}), k = 1, 2, 3, ..., n\} \).

6. Conduct a simple linear regression between \( \pi_1 k \) and \( p_1 k \). If the form of the prediction equation is correct, then the slope of the regression equation should be close to 1.0 and the intercept should be almost zero. Moreover, the coefficient of determination \( (r^2) \) should be close to 1.

APPLICATION OF DIMENSIONAL ANALYSIS AND SIMILITUDE METHODOLOGY TO SOIL-MACHINE SYSTEMS
<table>
<thead>
<tr>
<th>Authors</th>
<th>Topics</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>Barnes et al. (1960)</td>
<td>Simultaneity studies in tillage</td>
<td>Considered two approaches to representing soil properties. One model for soil considered specific weight of soil (FL⁻¹), moisture content (L), clay content (L), and soil-metal friction (L). This led to an undistorted model, but did not lead to good prediction of prototype disk draft (F). The second model considered cohesion (FL⁻¹), soil internal friction angle (L), adhesion (FL⁻¹), and soil-metal friction (L). This led to a distorted model. When the prediction factor was estimated from appropriate consideration of the distortion, the model predicted prototype behavior reasonably well. The other variables considered were: disk diameter (L), other pertinent lengths (L), angle of inclination (L), angle of approach (L), implement speed (LT⁻¹), and acceleration due to gravity (LT⁻²).</td>
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<tr>
<td>Hicks (1961)</td>
<td>Simultaneity studies in traction</td>
<td>Use of soil parameters in predicting the rolling resistance and sinkage of wheels was investigated. The author considered two dependent variables and 11 independent variables. Independent variables considered were: wheel diameter (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L), wheel load (L). The dependent variable was rolling resistance (F) and wheel sinkage (L).</td>
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<tr>
<td>Nuttall and McGowan (1961)</td>
<td>Simultaneity studies in traction</td>
<td>Reviewed the principles of dimensional analysis. Described simultaneity studies aimed at determining rolling resistance and sinkage of wheels. Considered two dependent and six independent variables. Independent variables considered were: wheel diameter (L), section height (L), slip (L), cohesion (FL⁻¹), internal frictional angle (L), and axle load (F). Dependent variables of interest were rolling resistance (F) and wheel sinkage (L).</td>
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<tr>
<td>Sullivan (1964)</td>
<td>Soil bin facilities for scale model</td>
<td>Modeled the weight of the soil in the scraper bowl as a function of ten independent variables. Independent variables considered were: size of the bowl (L), velocity of the bowl (LT⁻¹), cutting depth (L), duration of cutting (L), soil cohesion (FL⁻¹), internal angle of friction of soil (L), coefficient of soil-metal friction (L), specific weight of soil (FL⁻¹), and acceleration due to gravity (LT⁻²). The dependent variable was horizontal pushing force (F). Use of artificial soil, which was a mixture of clay, silt, and sand, for better control of soil properties and quicker preparation of test soil was emphasized.</td>
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<tr>
<td>Cline et al. (1963)</td>
<td>Scale model studies of off-road vehicles in clay soils</td>
<td>Considered two dependent and 14 independent variables to investigate the behavior of scale models of off-road vehicles to predict the performance of full-scale models in cohesive soils. Independent variables considered were: wheel diameter (L), tire width (L), tire deflection (L), forward speed of the vehicle (LT⁻¹), slip (L), pre-collapse structural cohesion (FL⁻¹), post-collapse structural cohesion (FL⁻¹), apparent structural cohesion (FL⁻¹), angle of internal friction (L), coefficient of friction between soil and the traction device (L), plastic kinematic viscosity of soil (FL⁻¹), specific weight of soil (FL⁻¹), soil moisture content (L), and vehicle weight (F). Dependent variables were: sinkage (L) and net traction (F). The results showed satisfactory predictions for drawer pull and poor predictions for sinkage. The authors attributed the poor results for sinkage to the inability to properly scale the soil.</td>
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<tr>
<td>Hegedus (1965)</td>
<td>Plate sinkage tests and dimensional</td>
<td>Considered one dependent and six independent variables. Independent variables considered were: characteristic length of plate (L), circumference of plate (L), specific weight of soil (FL⁻¹), cohesion (FL⁻¹), internal angle of friction (L), and applied pressure (FL⁻¹). The dependent variable was depth of sinkage (L). If cohesion was low or negligible, then distortion was minimal and the sinkage constant could be determined with the use of a single plate.</td>
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<tr>
<td>Colson (1965)</td>
<td>Artificial soils and soil bins for model testing of earthmoving equipment</td>
<td>Described the soil bin test facilities at Caterpillar Tractor, Inc., and use of artificial soils for model studies. Artificial soils were made by mixing fire clay, sand, and low-velocity oil to provide consistent engineering properties of soil (i.e., cohesion, adhesion, angle of internal friction, angle of soil-metal friction, and specific weight).</td>
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<tr>
<td>Freitag (1966)</td>
<td>Dimensional analysis of soil-treadless pneumatic tires in clay</td>
<td>Considered 11 independent and four dependent variables to investigate the performance of tireless pneumatic tires in clay. Independent variables selected were: tire diameter (L), section height (L), section width (L), deflection (L), cone index (FL⁻¹), spigot (FL⁻¹), load (F), wheel speed (LT⁻¹), slip (L), tire-soil friction coefficient (L), and acceleration due to gravity (LT⁻²). Dependent variables were: pull (F), force (FT), and sinkage (L). The concept of tire modality number was introduced.</td>
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<tr>
<td>Komayemi (1966)</td>
<td>Artificial soils for tillage studies</td>
<td>Used a mixture of clay, sand, and ethylene glycol to simulate natural soils with respect to cohesion and internal angle of friction.</td>
</tr>
<tr>
<td>Reeves (1966)</td>
<td>Artificial soils for tillage studies</td>
<td>A mixture of air-dried Houston clay and either spindle oil or ethylene glycol was used for making artificial soils of different specific weights. The effect of liquid content on cohesion, angle of internal friction, and soil-metal friction was investigated. A distorted model and a prototype plow chisel were tested in the artificial soil that used ethylene glycol as the fluid, which resulted in acceptable draft force prediction.</td>
</tr>
<tr>
<td>Young (1966, 1968, 1977)</td>
<td>Simulation of dynamic soil-machine interaction</td>
<td>Considered the application of dimensional analysis and characteristic equation in developing modeling laws. The formulation was done in terms of general variables. Applications of the technique to tillage and buried structure problems were discussed. The problems associated with distortion and ways to handle distortion in model studies were also outlined.</td>
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<td>Gurney et al. (1968)</td>
<td>Prediction of prototype performance from model studies</td>
<td>For geometrically similar bulldozers operating at low speed and same height of cut in similar soil type and condition, draft requirements were found to be functions of length scale only. Reeves et al. (1969) employed a more rigorous treatment on such data using similitude principles.</td>
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<tr>
<td>Freitag (1968)</td>
<td>Dimensional analysis of</td>
<td>Considered 13 independent and four dependent variables to investigate the performance of treadless...</td>
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<tr>
<td>Author(s)</td>
<td>Study Type</td>
<td>Variables and Findings</td>
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</table>
| Larson et al. (1968) | Simultaneous studies in tillage | Considered one dependent and 11 independent variables in the simulation study of moldboard plows. The independent variables were: tire diameter (L), section height (L), section width (L), deflection (L), cohesion (FL<sup>2</sup>), internal friction angle (L), specific weight (FL<sup>-1</sup>), spaghetti (FL<sup>-1</sup>T), load (F), wheel speed (LT<sup>-1</sup>), slip (-), tillage-soil friction (-), and acceleration due to gravity (LT<sup>-1</sup>). Dependent variables were: pull (F), torque (FL), and sinkage (L). For sandy soils, cone index gradient (FL<sup>-1</sup>) was used to replace specific weight of soil. The concept of sand number was discussed: 
\[ G = \frac{1}{\omega} \]
where \( G \) is cone index gradient with depth, \( \omega \) is section width, \( \delta \) is diameter, and \( W \) is axle load. |
<p>| Pierrot and Buchele (1968) | Simultaneous studies of unpowered pneumatic tire | Considered one dependent and ten independent variables in studying unpowered pneumatic tire performance. The independent variables were: tire diameter (L), tire load (F), tire configuration (( k_0 )), tire stiffness (FL&lt;sup&gt;-2&lt;/sup&gt;), wheel mass density (LT&lt;sup&gt;-2&lt;/sup&gt;F), sinkage plate penetration pressure (FL&lt;sup&gt;-2&lt;/sup&gt;), soil mass density (FL&lt;sup&gt;-2&lt;/sup&gt;T), speed (LT&lt;sup&gt;-1&lt;/sup&gt;), vertical load (F), and acceleration due to gravity (LT&lt;sup&gt;-1&lt;/sup&gt;). The dependent variable was rolling resistance (F). |
| Reeve et al. (1968) | Simultaneous in tillage studies | Considered one dependent and 12 independent variables in the simulation study of vertical chutes. The independent variables were: chisel width (L), depth (L), chisel angle (L), leading edge angle (L), angle of inclination (L), cohesion (ML&lt;sup&gt;-2&lt;/sup&gt;T&lt;sup&gt;-1&lt;/sup&gt;), internal friction angle (L), bulk density (NL&lt;sup&gt;-2&lt;/sup&gt;), adhesion (ML&lt;sup&gt;-2&lt;/sup&gt;T&lt;sup&gt;-1&lt;/sup&gt;), core index (ML&lt;sup&gt;-2&lt;/sup&gt;T&lt;sup&gt;-1&lt;/sup&gt;), chisel speed (LT&lt;sup&gt;-1&lt;/sup&gt;), and acceleration due to gravity (LT&lt;sup&gt;-1&lt;/sup&gt;). The dependent variable was draft force (MLT&lt;sup&gt;-2&lt;/sup&gt;). Distortion due to soil properties related to Pi term was considered, and the prediction factor was experimentally evaluated. |
| Schaffer et al. (1968) | Simultaneous in tillage studies | Considered one dependent and nine independent variables in simulation studies of disk-type implements. The independent variables were: disk diameter (L), other pertinent lengths (L), angle of approach (L), tilt angle (L), specific weight of soil (FL&lt;sup&gt;-1&lt;/sup&gt;), soil moisture content (L), other pertinent soil properties (FL&lt;sup&gt;-2&lt;/sup&gt;T&lt;sup&gt;-1&lt;/sup&gt;), disk speed (LT&lt;sup&gt;-1&lt;/sup&gt;), and acceleration due to gravity (LT&lt;sup&gt;-1&lt;/sup&gt;). The dependent variable was draft force (F). The models and the prototype were operated in the same soil condition, leading to distortion in the soil properties related to Pi term. Core index values and soil moisture content and its history were used to create similar soil conditions with the hope that all pertinent soil properties, which were not all known, were held constant during the experiments at desired levels. Distortion effect was considered, and the prediction factor was evaluated. |
| Reeve et al. (1969) | Simultaneous of bulldozer blades | Considered one dependent and five independent variables with the assumption that all relevant soil properties were held constant between models and the prototype during experiments, and blades had similar geometrical shape. The independent variables were: blade width (L), cutting depth (L), blade loading distance (L), operating speed (LT&lt;sup&gt;-1&lt;/sup&gt;), and acceleration due to gravity (LT&lt;sup&gt;-1&lt;/sup&gt;). The dependent variable was draft force (F). Concept of load-growth curve was used to accurately predict draft requirements of bulldozer blades. |
| Schaffer et al. (1969) | Simultaneous in soil-machine systems and distortion due to instability in scaling soil properties | Considered a simplified case in which only one dependent and five independent variables were considered for a soil-machine system that involved the operation of both models and prototype in the same soil type and condition at a low ground speed. The variables considered were: characteristic length (L), other pertinent lengths (L), desired force (F), other relevant forces (F), characteristic soil property (FL&lt;sup&gt;-1&lt;/sup&gt;), and other pertinent soil properties (FL&lt;sup&gt;-1&lt;/sup&gt;). The authors showed that the prediction factor was a single power function of the length scale used. The existence of this relationship makes the use of distorted models a valuable research tool in model studies. |
| Freitag et al. (1970a, 1970b, 1977) | Simultaneous studies of soil-machine system | State-of-the-art review paper related to simulation in soil-machine system. Detailed accounts of relevant soil properties, their measurement, inconsistencies in measured values due to different techniques, desirability of using analog devices to measure soil properties, difficulty in scaling soil properties, problems in maintaining same soil properties for model and prototype even when soil preparation techniques were the same, and use of artificial soils were given. |
| Sprinkle et al. (1970) | Simulation study with static and dynamic properties of artificial soils | Considered different combination of variables to predict the draft of rectangular blades using scale models. Use of artificial soils (Goose lake fire clay and SAE SW oil) to scale soil properties was investigated. Moreover, scaling for speed effect was also found to be important. Inclusion of various effects (speed effect) improved the prediction at higher speeds. At lower speeds (~1.6 kmh or 1 mph), soil cohesion was the main property that influenced soil cutting force. Artificial soils could be used to properly scale cohesion. |
| Wang and Liang (1970) | Tillage tool draft | Outlined a technique to predict draft of a tillage tool in any soil and at any speed using a geometrically similar model. However, the technique required selecting length scale based on soil properties (cohesion and specific weight of soil). The investigators considered eight variables. Independent variables were: characteristic length (L), cohesion (FL&lt;sup&gt;-2&lt;/sup&gt;), internal friction angle (L), specific weight (FL&lt;sup&gt;-1&lt;/sup&gt;), tillage tool speed (LT&lt;sup&gt;-1&lt;/sup&gt;), apparent soil-tool friction angle (L), and acceleration due to gravity (LT&lt;sup&gt;-1&lt;/sup&gt;). The dependent variable was draft force (F). |</p>
<table>
<thead>
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<th>Reference</th>
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<tr>
<td>Luth and Wisner (1971)</td>
<td>Development of prediction equations for soil cutting blades in sand using dimensional analysis. Considered two dependent and seven independent variables to describe soil cutting by blades since soil metal friction and soil internal friction angle were constant for sandy soils used in these studies. Independent variables were: blade width (L), blade length (L), operating depth (L), blade angle (°), specific weight of soil (FL°), operating speed (LT°), and acceleration due to gravity (LT²). Dependent variables were: draft force (F) and vertical reaction (Fv).</td>
</tr>
<tr>
<td>Verma and Schafer (1971a)</td>
<td>Compensation of distortion in soil-machine studies. Developed a general technique to compensate for distortion due to soil properties (i.e., operating model and prototype in the same soil). The procedure was demonstrated by considering a soil-prototype system consisting of one dependent variable and six independent variables. Independent variables were: chisel width (L), leading apex angle (°), angle of inclination (°), depth of cut (L), a characteristic soil property (FL°), and other pertinent soil properties (FL°). The dependent variable was draft force (F). A geometric Pi term (depth of cut/chisel width) was distorted to compensate for the distortion due to soil properties when the prototype and models were operated in the same type of soil and condition. Reasonably good prediction was achieved using this technique in soil-prototype studies.</td>
</tr>
<tr>
<td>Verma and Schafer (1971b)</td>
<td>Distorted models in non-uniform soil profiles. Developed a technique to compensate for distortion due to the operation of prototype and models at the same depth in non-uniform soils with non-uniformity in soil profile was modeled. The methodology was demonstrated using a soil-prototype system. The variables considered were the same as in Verma and Schafer (1971a). They showed that operation of the models and prototype at the same depth in soils with non-uniform profiles was a useful technique to deal with non-uniformity of soil profile with depth.</td>
</tr>
<tr>
<td>Schaefer et al. (1972, 1977)</td>
<td>Distortion in the simulation studies of soil-machine systems. State-of-the-art review paper that addresses the issue of distortion in simulation studies of soil-machine systems. It lists various sources of distortion and the role played by soil properties in inducing distortion. Various cases involving purely frictional, purely cohesive, and cohesive-frictional soil were considered. Techniques for dealing with distorted models were described in detail.</td>
</tr>
<tr>
<td>Wisner and Luth (1972)</td>
<td>Development of prediction equations for soil cutting blades using dimensional analysis. Considered two dependent and nine independent variables to describe soil cutting by blades in almost fully saturated clay. Independent variables were: blade width (L), blade length (L), operating depth (L), blade angle (°), specific weight of soil (FL°), cohesion (FL°), shear rate factor (°), operating speed (LT°), and acceleration due to gravity (LT²). Dependent variables were: draft force (F) and vertical reaction (Fv). Reasonably good prediction was achieved using these techniques in soil-prototype studies. The soil internal angle of friction was negligible, and soil-metal friction was constant. Soil cohesion and shear rate effect were incorporated using a dimensionless term developed from cone index values obtained using a standard cone blade speed and blade width.</td>
</tr>
<tr>
<td>Swanson (1973a)</td>
<td>Dimensional analysis to develop prediction equations for performance of dual and tandem rigid wheels in sand. Considered two dependent and seven independent variables in the study of performance of dual and tandem rigid wheels in sand. Independent variables were: wheel diameter (L), wheel width (L), dual wheel spacing (L), frictional soil sinkage parameter (FL°), soil sinkage exponent (n), and load (F). Dependent variables were: tire deflection (L), tire deflection (L), and load (F).</td>
</tr>
<tr>
<td>Swanson (1973b)</td>
<td>Scale model studies of test tire tests in clay. Tests with scale model tire tests were conducted to verify the use of a slightly modified dimensionless clay mobility number (Nc) in predicting the performance of test tires in clay.</td>
</tr>
<tr>
<td>Swanson and Luth (1973, 1974)</td>
<td>Off-road vehicle traction prediction equations using dimensional analysis. A set of simple, widely used traction prediction equations for off-road vehicles was developed using three dependent and six independent variables. Independent variables were: tire section width (L), overall tire diameter (L), rolling radius (L), soil core index value (FL°), axle load (F), and wheel slip (°). Dependent variables were: tire deflection (F), tire deflection (F), and load (F). The concept of wheel moment was used to represent soil type and condition.</td>
</tr>
<tr>
<td>Burt et al. (1974a)</td>
<td>Similar studies related to traction. Considered one dependent and seven independent variables to predict the dynamic net traction of smooth, rigid wheels operating at relatively low speed. Independent variables were: wheel diameter (L), wheel width (L), other pertinent length parameters (L), slip (°), characteristic soil property (FL°), other pertinent soil properties (FL°), and dimensionless soil properties (n). The dependent variable was dynamic net traction (F). The authors did not specify the soil properties but wanted to determine the dimension of the relevant soil property or properties based on the study of the distortion factor. They found fundamental differences between large prototype and small models that made it impossible to determine the dimensions of the relevant soil property or properties.</td>
</tr>
<tr>
<td>Burt et al. (1974b)</td>
<td>Similar studies related to traction. Considered one dependent and seven independent variables to predict the sinkage of smooth, rigid wheels operating at relatively low speed. The independent variables were: wheel diameter (L), wheel width (L), other pertinent length parameters (L), slip (°), characteristic soil property (FL°), other pertinent soil properties (FL°), and dimensionless soil properties (n). The dependent variable was wheel sinkage (L). Similar to Burt et al. (1974a), the authors did not specify the soil properties but wanted to determine the dimension of the relevant soil property or properties based on the study of the distortion factor. They found fundamental differences between large prototype and small models. However, the difference in the fundamental behavior was not as significant as it was for the prediction of dynamic net traction. The dimension of the relevant soil property was found to be FL°.</td>
</tr>
<tr>
<td>Wisner et al. (1976)</td>
<td>Application of similarity to soil-machine systems. Provides a state-of-the-art review of dimensional analysis and similarity, and discusses the application of these principles to pneumatic tires in soft soils, soil cutting, tillage implements, and bulldozers.</td>
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<tr>
<td>Author(s)</td>
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<td>Wismer et al.</td>
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<td>Geng-Shou et al.</td>
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<td>Pandey and Ojha</td>
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<td>Brixius</td>
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<td>Sato and Ninh</td>
<td>1993</td>
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<td>Camilas and Salokhe (2001, 2002)</td>
<td>Prediction equation development using dimensional analysis</td>
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LESSON 15. Development of prediction equations

The requirement that a general homogeneous equation is dimensionally consistent makes it possible to develop prediction equations as follows:

Consider the gravitational component of stress within a homogeneous soil mass at a depth \(d\) below the surface in the absence of surcharge. Let \(\sigma\) be the stress and \(\gamma\) be the specific weight. Then the stress \(\sigma\) is given by:

\[
\sigma = f(d, \gamma) = C d^{C_1} \gamma^{C_2}
\]

or, in terms of dimensions:

\[
\begin{align*}
\left\{ \text{stress}\right\} & = F \left\{ L^{-2} \right\} d \quad \left\{ \text{length}\right\} \\
\left\{ \text{specific weight}\right\} & = F \left\{ L^{-3} \right\}
\end{align*}
\]

where \(C\) is a dimensionless constant, and \(\left[\dot{\text{dot}} = \right]\) represents dimensional equivalence and not necessarily numerical equivalence. Therefore:

\[
\begin{align*}
\left\{ \sigma \right\} & = F \left\{ L^{-2} \right\} d \\
\left\{ \gamma \right\} & = F \left\{ L^{-3} \right\}
\end{align*}
\]

Since the dimensions must be consistent for this equation to be general, exponents of each dimension should match, i.e.:

\[
\begin{align*}
1 & = C_2 \quad \text{(2.4a)} \\
-2 & = C_1 - 3C_2 \quad \text{(2.4b)}
\end{align*}
\]

From equations 2.4a and 2.4b, we have \(C_1 = 1\) and \(C_2 = 1\). Therefore, equation 2.1 becomes:

\[
\sigma = C \gamma d = C \gamma d
\]

In this case, dimensional analysis yielded the form of the equation completely, except for the dimensionless constant \(C\). This constant, which is equal to unity (i.e., \(C = 1\), can be determined by conducting limited experiments.

Now consider the case of a wide vertical blade (i.e., rake angle is 90°) of width \(w\), operating at depth \(d\), in a cohesionless, homogeneous soil in the absence of any surcharge. The draft \((D)\) needed to overcome the gravitational component of the soil cutting resistance (i.e., the soil cutting force necessary to resist the passive pressure on the tillage tool due to the soil weight) under quasistatic conditions (i.e., very low ground speed) is dependent on the specific weight of the soil \((\gamma)\), operating depth \((d)\), blade width \((w)\), and soil internal friction angle \((\varphi)\) in the absence of soil metal friction. Therefore, we have:

\[
D = f(d, w, \gamma, \varphi) = C d^{C_1} w^{C_2} \gamma^{C_3} f_{C_4}
\]
or, in terms of dimensions:

\[ D \dot{=} F \]
\[ d \dot{=} L \]
\[ w \dot{=} L \]
\[ \gamma \dot{=} FL^{-3} \]
\[ \phi \text{ dimensionless} \]

Therefore, in terms of dimensions, equation (2.6) becomes:

\[ F \dot{=} (L)^{C_1}(L)^{C_2}(FL^{-3})^{C_3} \]  \hspace{1cm} (2.7)

Or,

\[ F \dot{=} (L)^{C_1+C_2-3C_3}(F)^{C_3} \]  \hspace{1cm} (2.8)

Therefore,

\[ C_1 + C_2 - 3C_3 = 0 \]  \hspace{1cm} (2.9a)
\[ C_3 = 1 \]  \hspace{1cm} (2.9b)

From equations (2.9a) and (2.9b), we get:

\[ C_1 + C_2 = 3 \text{ or } C_2 = 3 - C_1 \]  \hspace{1cm} (2.9c)

Therefore, from equation (2.6), we have:

\[ D = Cd_1w^{3-C_1}\gamma f^{C_3} \]  \hspace{1cm} (2.10)

Simplifying and rearranging equation (2.10), we obtain:

\[ \frac{D}{\gamma w^3} = C \left( \frac{d}{w} \right)^{C_1} \phi^{C_4} \]  \hspace{1cm} (2.11)

Although equation (2.11) provides the form, constants \( C, C_1, \) and \( C_4 \) are unknown and should be determined experimentally. Note that \( (D/\gamma w^3), (d/w), \) and \( \phi \) are all dimensionless terms and are often called Pi terms. A more general way of writing equation (2.11) is to express the dependent Pi term \( (D/\gamma w^3) \) as a function of independent Pi terms \( (d/w) \) and \( \phi \) (Murphy, 1950). Thus, equation (2.11) can be written in a more general form as:

\[ \frac{D}{\gamma w^3} = \Phi \left( \frac{d}{w}, \phi \right) \]  \hspace{1cm} (2.12)
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In going from equation (2.6) to equation (2.11), a five-variable problem has been reduced to a three-variable or $\Pi$ terms problem. This reduction in the number of variables is a major advantage provided by dimensional analysis. This reduction in the number of variables is easily determined by the Buckingham $\Pi$ theorem.
LESSON 16. Methods of dimensional analysis

The following two methods of dimensional analysis are generally used.

i. Rayleigh Method

ii. Buckingham p-Method

16.1 Rayleigh Method

As early as 1899, Lord Rayleigh made an ingenious application of dimensional analysis to the problem of the effect of temperature on the viscosity of gas. Rayleigh’s method is outwardly different from Buckingham’s method; let us consider the drag force \( F \) that a smooth spherical body experiences in a stream of incompressible fluid. Consider tentatively a relationship of the form,

\[
F = V^a D^b r^c m^d
\]

The exponents must be adjusted to make the equation dimensionally homogeneous. This leads to the more special form

\[
F = r V^2 D^2 R^n
\]

Where \( R = V D r/m \) and \( n \) is a numerical exponent.

Equation (3.1) is of such of restricted form that can be expected to represent the phenomenon. However, Rayleigh pointed out that special solutions of the type of the equation b may be summed to give more general solutions. Accordingly a general type of dimensionally homogeneous relationship is

\[
F = r V^2 D^2 A_n R^n
\]

In which the coefficients \( A_n \) are dimensionless constants. Since the series is a general function of \( R \), the solution is of the form,

\[
F = r V^2 D^2 f(R)
\]

in which \( f \) is an unspecified function.

16.2 Buckingham’s Pi theorem

Evidently, any equation that relates dimensionless products is dimensionally homogeneous; i.e. the form of the equation does not depend on the fundamental units of measurement. This observation may be formally stated as follows:
A sufficient condition that an equation be dimensionally homogeneous is that it be reducible to an equation among dimensionless products.

E. Buckingham inferred the fundamental principle that the conditions of this theorem are also necessary. Buckingham’s theorem is accordingly stated as follows:

‘If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products’.

In other words, “Any equation that relates dimensionless product is dimensionally homogeneous. The condition that an equation be dimensionally homogeneous is to an equation among dimensionless products”.

This theorem is, by no means, self-evident. Buckingham himself did not rigorously prove the theorem, although he presented evidence to make its truth seem plausible.

Buckingham’s theorem summarizes the entire theory of dimensional analysis. However, principles of dimensional analysis were employed before this theorem was expounded.

Let us now consider the problem in 3.1 in the light of Buckingham’s theorem. We assume only that the five variables are related by a dimensionally homogeneous equation. This may be indicated by \( f(F,V,D,\rho,\mu) = 0 \), in which \( f \) is an unspecified function.

The Buckingham’ theorem states that if there are \( n \) dimensional variables involved in a phenomenon, which can be completely described by \( m \) fundamental quantities or dimensions (such as mass, length, time etc.,) and are related by a dimensionally homogeneous equation, then the relationship among the \( n \) quantities can always be expressed in terms of exactly \( (n-m) \) dimensionless and independent \( \pi \) terms.

**16.2.1 Procedure for Buckingham’s Pi method**

a. List all the ‘\( n \)’ physical quantities or variables involved in the phenomenon. Note their dimensions and the number ‘\( m \)’ of the fundamental dimensions comprised in them. So that there will be \( (n-m) \). \( \pi \) terms.

b. Select ‘\( m \)’ variables out of these to serve as repeating variables with the following guidelines:

i. These variables should be such that none of them is dimensionless.
ii. No two variables have the same dimensions.
iii. They themselves do not form a dimensionless parameter.
iv. The entire ‘\( m \)’ fundamental are included collectively in them.
v. The dependent variable should not be taken as repeating variable.

c. Write the general equations for different \( \pi \) terms. These may be expressed as the product of the repeating variables each raised to an unknown exponent and one of the remaining variables taken in turn, with a known power (usually taken as one).

d. Write the dimensional equations of the equations of the \( \pi \) terms obtained in the step (c) above.
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Compute the value of the unknown exponents by equating the exponents of the respective fundamental dimensions on both the sides of each of the dimensional equations. Thereby different dimensional groups or π terms are formed.

e. Write the final general equation for the phenomenon in terms of the π terms.

In order to obtain the final expression the following additional may be considered

i. If the quantity is dimensionless, it is a π term without going through the above procedure.

ii. If any two physical quantities have the same dimensions, their ratio will be one of the π term. For example \((H/d)\) is dimensionless and hence it is a π term.

iii. Any π term may be replaced by any power of that term, including negative as well as fractional powers. For example, \(\pi_1\) may be replaced by \(\pi^1\), or \(\pi_2\) may be replaced by \(\pi^2\), or \(\pi_3\) may be replaced by \(\frac{1}{\pi^2}\), etc.

iv. Any π term may be replaced by multiplying it by numerical constant. For example, \(\pi_1\) may be replaced by \(3\pi_1\) or so.

v. Any π term may be replaced by another π term obtained by adding or subtracting an absolute numerical from it.

vi. Any π term may be replaced by multiplying it by another π term. For example, \(\pi_1\) may be replaced by \((\pi_1 \times \pi_2)\).

Mathematically, if any variable \(Q_1\) depends on the independent variables \(Q_2, Q_3, Q_4, \ldots Q_n\); the fundamental equation may be written as,

\[ Q_1 = f(Q_2, Q_3, Q_4, \ldots Q_n) \]

which can be transformed to another functional relationship as,

\[ f_1(Q_1, Q_2, Q_3, Q_4, \ldots Q_n) = C \]

where ‘C’ is the dimensionless constant.

In accordance with the π theorem, a non-dimensional equation can thus be obtained in the form,

\[ f_2(\pi_1, \pi_2, \pi_3, \ldots \pi_{n-m}) = C_1 \]

wherein, each dimensionless π-term is formed by combining \(m\) variables out of the total \(n\) variables with one of the remaining \((n-m)\) variables. These ‘\(m\)’ variables which appear repeatedly in each of the π terms, are called repeating variables. These are ‘\(m\)’ fundamentals quantities. They themselves do not form a dimensionless parameter. Thus the different π terms may be established as,

\[ \pi_1 = Q_1^{a_1}, Q_2^{b_1}, Q_3^{c_1}, \ldots Q_{n-m_1}, Q_{m+1} \]

\[ \pi_2 = Q_1^{a_2}, Q_2^{b_2}, Q_3^{c_2}, \ldots Q_{n-m_2}, Q_{m+2} \]
In the above equation, each individual equation is dimensionless and the exponents $a, b, c, d \ldots \ldots m$ etc are determined by considering dimensional homogeneity for each equation such away that each $\pi$ term is dimensionless.

The final general equation for the phenomenon may then be obtained by expressing any one of the $\pi$ terms as a function of the others.

$$\pi_1 = f_1 (\pi_1, \pi_2, \pi_3, \ldots \ldots \pi_{n-m})$$

$$\pi_2 = f_2 (\pi_1, \pi_2, \pi_3, \ldots \ldots \pi_{n-m})$$

or any other desired relationship may be obtained.

The use of Buckingham’s Pi method is illustrated by an example.

Consider the problem of drag force that a smooth spherical body experiences in a stream of compressible fluid. Assume that five variables namely, drag force ($F$), velocity ($V$), diameter of sphere ($D$), density of fluid ($\rho$) and absolute viscosity ($\mu$), are related by a dimensionally homogeneous equation. This may be indicated by

$$F = f(V, D, \rho, \mu).$$

This may be written in the general form

$$f_1 (F, V, D, \rho, \mu) = C$$

The total number of variables, $n = 5$

These variables may be completely described by three fundamental dimensions of $M - L - T$ ($m = 3$ fundamental unit).

Therefore the number of $\pi$ terms = $n-m = 5 - 3 = 2$ (i.e., Number of $\pi$ terms is 2)

$$f_2 (\pi_1, \pi_2) = C_1$$

In order to form these $\pi$ terms, choose the three repeating variables following the guidelines, since the fundamental dimensions are three. Choose $r, V$ and $D$ as repeating variables. Since physical quantities of the similar dimensions can neither be added nor subtracted the terms are expressed as products as follows:

$$\pi_1 = r^a V^b D^c m$$  \hspace{1cm} (3.3)

$$\pi_2 = r^e V^d D^f F$$  \hspace{1cm} (3.4)

Expressing $\pi_1$ dimensionally in terms of $M - L - T$, we get

$$\left( \frac{M}{L^3} \right)^a \left( \frac{L}{T} \right)^b L^c m = M^0 L^0 T^0$$
Equating the exponents of $M$, $L$, and $T$, we get

For $M$: \[ a_1 + 1 = 0; \quad a_1 = -1 \]  \hspace{1cm} (3.5)

For $L$: \[ -3 a_1 + b_1 + c_1 - 1 = 0 \]  \hspace{1cm} (3.6)

For $T$: \[ -b_1 - 1 = 0; \quad b_1 = -1 \]  \hspace{1cm} (3.7)

Substituting the value of $a_1$ & $b_1$ in equation 3.5 and $e$ in equation 3.6, we get

\[ 3-1 + c_1 - 1 = 0: \quad c_1 = -1 \]

Substituting the values of $a_1$, $b_1$, and $c_1$ in equation 3.3, we get

\[ \pi_1 = \frac{\mu}{\rho V D} \quad \text{or} \quad \frac{\rho V D}{\mu} \]

Similarly,

\[ \pi_2 = \rho c^2 V^2 D^2 \]

Expressing $\pi_1$ dimensionally in terms of $M$-$L$-$T$, we get

\[ \left( \frac{M}{L^3} \right)^{a_2} \left( \frac{L}{T} \right)^{b_2} \left( L \right)^{c_2} \]  \hspace{1cm} (3.8)

\[ a_2 + 1 = 0; \quad a_2 = -1 \]

For $L$: \[ -3 a_2 + b_2 + c_2 + 1 = 0 \]  \hspace{1cm} (3.9)

For $T$: \[ -b_2 - 2 = 0; \quad b_2 = -2 \]  \hspace{1cm} (3.10)

Substituting the value of $a_2$ & $b_2$ in equation 3.8 and 3.10 in equation 3.9, we get

\[ 3-2 + c_2 + 1 = 0: \quad c_2 = -2 \]

Substituting the values of $a_2$, $b_2$, and $c_2$ in equation 3.4, we get

\[ \pi_2 = \frac{F}{\rho V^2 D^2} \]

Buckingham’s theorem asserts that, since the equation is dimensionally homogeneous, $f$ is not actually a function of five separate variables, but rather a function of a complex set of dimensionless products of the variables is comprised of the pressure coefficient, $P = Fp^1 V^{-2} D^{-2}$, and the Reynolds number, $R = V D p^{-1}$. Hence, by Buckingham’s theorem, the equation is reducible to the form, $f \left( P, R \right) = 0$. This relationship may be indicated in the explicit form, $P = f(R)$. If $f$ is regarded merely as a symbol for some function, the relationships, $f \left( P, R \right) = 0$ and $P = f(R)$, mean the same thing; namely, that is possible to plot a curve that shows the relationship between $P$ and $R$. The equation, $P = f(R)$, is same result that was
obtained above, by Rayleigh’s method. The reasoning that has led to the conclusion, $P = f(R)$, is not restricted to spherical bodies, it is valid for a body of any shape. Example: airplane wing. The form of the curve that relates $P$ to $R$ depends, of course, on shape of the body. Dimensional analysis provides no information concerning the form of the curve.

Rayleigh’s method of dimensional analysis does not differ intrinsically from Buckingham’s method. The algebraic steps in two methods are essentially same. However, Buckingham’s method absolves us from the indiscriminate use of infinite series. Too often, it is not explained that the construction of an infinite series is a logically indispensable step in Rayleigh’s method. Consequently, the impression is created that the dependent variable in a physical problem may be arbitrarily equated to a product of powers of the independent variables and a numerical coefficient. This assumption is sometimes a legitimate approximation, particularly in problems of heat transfer, but it is not essential part of dimensional analysis.

If $n$ variables are connected by unknown dimensionally homogeneous equation, Buckingham’s theorem allows up to conclude that the equation can be expressed in the form of a relationship among $(n - m)$ dimensionless products, in which $(n - m)$ is the number of products in a complex set of dimensionless products of the variables. In most cases, $m$ is equal to the number of fundamental dimensions in the problem. However, this cannot be an infallible rule, since the number of fundamental dimensions in a problem may depend on the system of fundamental dimensions that is used.
**LESSON 17. Application of dimensional analysis and similitude to soil mechanics**

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<td>Bames et al, (1960)</td>
<td>Similitude studies in tillage</td>
<td>Considered two approaches to representing soil properties. One model for soil considered specific weight of soil (FL), moisture content (-), clay content (-), and soil-metal friction (-). This led to an undistorted model, but did not lead to good prediction of prototype disk draft (F). The second model considered cohesion (FL(^{-2})), soil internal friction angle (-), adhesion (FL(^{-2})), and soil-metal friction (-). This led to a distorted model. When the prediction factor was estimated from appropriate consideration of the distortion, the model predicted prototype behavior reasonably well. The other variables considered were: disk diameter (L), other pertinent lengths (L), angle of inclination (-), angle of approach (-), implement speed (LT(^{-1})), and acceleration due to gravity (LT(^{-2})).</td>
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<tr>
<td>Hicks (1961)</td>
<td>Similitude studies in traction</td>
<td>Use of sinkage parameters in predicting the rolling resistance and sinkage of wheels was investigated. The author considered two dependent variables and 11 independent variables. Independent variables considered were: wheel width (L), wheel diameter (L), soil depth (L), wheel aspect ratio (-), cohesive sinkage modulus (FL(^{-(n+2)})), frictional sinkage modulus (n(FL^{-(n+2)})), sinkage exponent (-), coefficient of soil-tire friction (-), specific weight of soil (FL(^3)), and acceleration due to gravity (LT(^{-2})). Dependent Variables were: rolling resistance (F) and wheel sinkage (L).</td>
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<tr>
<td>Nuttall and McGowan (1961)</td>
<td>Similitude studies, in traction</td>
<td>Reviewed the principles of dimensional analysis, Described similitude studies aimed at determining rolling resistance and sinkage of wheels. Considered two dependent and six independent variables. Independent variables considered were: wheel diameter (L), section height (L), slip (-), cohesion (FL(^{-2})) internal frictional angle (-), and axle load (F). Dependent variables of interest were rolling resistance (F) and wheel sinkage (L).</td>
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<tr>
<td>Sullivan (1964)</td>
<td>Soil bin facilities for scale model studies at Caterpillar Tractor, Inc and application of similitude in</td>
<td>Modeled the weight of the soil in the scraper bowl as a function of independent variables. Independent variables considered were: size of the bowl (L), velocity of the bowl (LT(^{-2})), cutting depth (L), duration of cutting (T), soil cohesion (FL(^{-2})), internal angle of friction of soil (-), coefficient of soil-metal friction (-), specific weight of soil (FL(^3)), and acceleration due to gravity (LT(^{-2})). The dependent variable was horizontal pushing force (F). Use of artificial soil, which was a mixture of clay, silt, and sand, for better control of soil properties and quicker control of soil properties.</td>
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<tr>
<td>Study</td>
<td>Scope</td>
<td>Results/Findings</td>
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<tr>
<td>Clark et al. (1965)</td>
<td>Scale model studies of off-road vehicles in clay soil</td>
<td>Considered two dependent and 14 independent variables to investigate the behavior of scale models of off-road vehicles to predict the performance of full-scale models in cohesive soils. Independent variables considered were: wheel diameter (L), tire width (L), tire deflection (L), forward speed of the vehicle (LT⁻¹), slip (-), pre-collapse structural cohesion (FL⁻²), post-collapse structural cohesion (FL⁻²), apparent structural cohesion (FL⁻²), angle of internal friction (-), coefficient of friction between soil and the traction device (-), plastic kinematic viscosity of soil (FL⁻²T), specific weigh of soil (FL⁻³), soil moisture content (-) and vehicle weight (F). Dependent variables were: sinkage (L) and net traction (F). The results showed satisfactory predictions for drawbar pull and poor correlations for sinkage. The authors attributed the poor results for sinkage to the inability to properly scale the soil.</td>
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<tr>
<td>Hegedus (1965)</td>
<td>Plate sinkage test and dimensional analysis</td>
<td>Considered one dependent and six independent variables, Independent variables considered were: characteristic length of plate (L), circumference of plate (L), specific weight of soil (FL⁻²), cohesion (FL⁻²), internal angle of friction (-), and applied pressure (FL⁻¹). The dependent variable was depth of sinkage (L). If cohesion was low or negligible then distortion was minimal and the sinkage constant could be determined with the use of a single plate.</td>
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<tr>
<td>Cohron (1966)</td>
<td>Artificial soils and soil bins for model testing of earthmoving equipment</td>
<td>Described the soil bin test facilities at Caterpillar Tractor, Inc., and use of artificial soils for model studies. Artificial soils were made by mixing fire clay, sand, and low-viscosity oil to provide consistent engineering properties of soils (i.e., cohesion, adhesion, angle of internal friction, specific weight).</td>
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<tr>
<td>Freitag (1966)</td>
<td>Dimensional analysis of soil-treadless pneumatic tires in clay</td>
<td>Considered 11 independent and four dependent variables to investigate the performance of treadless pneumatic tires in clay. Independent variables selected were: tire diameter (L), section height (L), section width (L), deflection (L), cone index (FL⁻²), spissitude (FL⁻¹T), load (F), wheel speed (LT⁻¹), slip (-), tire-soil friction (-), and acceleration due to gravity (LT⁻¹). Dependent variables were: pull (F), towed force (F), torque (FL), and sinkage (L). The concept of clay mobility number was introduced;</td>
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</table>
where C is the cone index value, $W$ is axle load, $b$ is the section width of tire, $d$ is the overall diameter, $h$ is the section height, and the tire deflection.

<table>
<thead>
<tr>
<th>Korayem (1966)</th>
<th>Artificial soils for tillage studies</th>
<th>Used of a mixture of day, sand, and ethylene glycol to simulate natural soils with respect to cohesion and internal angle of friction.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaves (1966)</td>
<td>Artificial soils for tillage studies</td>
<td>A mixture of air-dried Houston clay and either spindle oil or ethylene glycol was used for making artificial soils of different specific weights. The effect of liquid content on cohesion, angle of internal friction, and soil-metal friction was investigated. A distorted model and a prototype plane chisel were tested in the artificial soil that used ethylene glycol as the fluid, which resulted in acceptable draft force prediction.</td>
</tr>
<tr>
<td>Young (1966, 1968, 1977)</td>
<td>Similitude of dynamic soil-machine interaction</td>
<td>Considered the application of dimensional analysis and characteristic equation in developing modeling laws. The formulation was done in terms of general variables. Applications of the technique to tillage and buried structure problems were discussed. The problems associated with distortion and ways to handle distortion in model studies were also outlined.</td>
</tr>
<tr>
<td>Garrity et al. (1968)</td>
<td>Prediction of prototype performance from model studies</td>
<td>For geometrically similar bulldozer blades operating a low speed and same height of cut in similar soil type and condition, draft requirements were found to be functions $f$ length scale only. Reaves et al. (1969) employed a more rigorous treatment such data using similitude principle</td>
</tr>
<tr>
<td>Freitag (1968)</td>
<td>Dimensional analysis of soil-treadless pneumatic tires in clay</td>
<td>Considered 13 independent and four dependent variables to investigate the performance of treadless pneumatic tires in sand. The independent variables were, tire diameter ($L$), section height ($L$) section width ($L$), deflection ($L$) cohesion ($FL^{-2}$) internal friction angle ($\cdot$), specific weight ($FL^{-3}$), spissitude ($FL^{-2}T$), load ($F$), wheel speed ($LT^{-1}$), slip ($\cdot$), tire-soil friction ($\cdot$), and acceleration due to gravity ($LT^{-2}$). Dependent variables were: pull ($F$), towed force ($F$), torque ($FL$), and sinkage ($L$). For sandy soils, cone index gradient ($FL^{-3}$) was used to replace specific weight of</td>
</tr>
</tbody>
</table>
The concept of sand number was discussed:

\[
\frac{G(bd)^{3/2}}{W}
\]

Where \( G \) is cone index gradient with depth \( b \) is section width, \( d \) is diameter, and \( W \) is axle load.

<p>| <strong>Larson et al. (1968)</strong> | Similitude studies in tillage | Considered one dependent and 11 independent variables in the similitude study of moldboard plows. The independent variables were: plow width (L), all other pertinent lengths (L), lateral angle of plow surface (-), a design parameter related to the shape of the moldboard (-), cohesion (FL^{-2}), internal friction angle (-), specific weight of soil (FL^{-3}), adhesion (FL^{-2}), soil-metal friction angle (-), implement speed (LT^{-1}), and acceleration due to gravity (LT^{-2}). The dependent variable was draft force (F). Distortion arising from cohesion and internal friction angle related Pi terms were found to be important, and prediction factors were developed for overcoming distortion caused by these variables. Effects of disorientation due to Pi terms related to adhesion and soil-metal friction were found to be negligible. |
| <strong>Pierrot and Buchele (1968)</strong> | Similitude studies of unpowered pneumatic tire | Considered one dependent and ten independent variables in studying unpowered pneumatic tire performance. The independent variables were: tire diameter (L), width (L), tread configuration (( \lambda )), tire stiffness (FL^{-1}), wheel mass density (FL^{-4}T^{-2}), sinkage plate penetration pressure (FL^{-2}), soil mass density (FL^{-4}T^{-2}), speed (LT^{-2}), vertical lead (F), and acceleration due to gravity (LT^{-2}). The dependent variable was rolling resistance (F). |
| <strong>Reaves et al. (1968)</strong> | Similitude studies in tillage | Considered one dependent and 12 independent variables in the similitude study of vertical chisels. The independent variables were: chisel width (L), depth (L), surface roughness (L), leading apex angle (-), angle of inclination(-), cohesion (ML^{-1}T^{-2}), internal friction angle (-), bulk density (ML^{-2}), adhesion (ML^{-1}T^{0}), cone index (ML^{-1}T^{-2}), chisel speed (LT^{-1}), and acceleration due to gravity (LT^{-2}). The dependent variable was draft force(ML^{-1}T^{2}). Distortion due to soil properties related Pi term was considered, and the prediction factor was experimentally evaluated. |
| <strong>Scafer et al. (1968)</strong> | Similitude in tillage studies | Considered one dependent and nine independent variables in similitude studies of disk-type implements. The independent variables were: disk diameter (L), all other pertinent lengths (L), angle of |</p>
<table>
<thead>
<tr>
<th>Study Authors</th>
<th>Study Details</th>
<th>Variables Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaves et al. (1969)</td>
<td>Similitude in bulldozer blades</td>
<td>approach (-), tilt angle (-), specific weight of soil (FL⁻³), soil moisture content (-), other pertinent soil properties (F⁻²L⁻⁴T⁻¹), disk speed (LT⁻²) and acceleration due to gravity (LT⁻²). The dependent variable was draft force (F). The models and the prototype were operated in the same soil condition leading to distortion in the soil properties related Pi term, Cone index values and soil moisture content and its history were used to create similar soil conditions with the hope that all pertinent soil properties, which were not all known, were held constant during the experiments at desired levels. Distortion effect was considered, and the prediction factor was evaluated.</td>
</tr>
<tr>
<td>Scafer et al. (1968)</td>
<td>Similitude in soil-machine systems and distortion due to inability in scaling properties</td>
<td>Considered one dependent and five independent variables With the assumption that all relevant soil properties were held constant between models and the prototype during experiments, and blades had similar geometrical shape. The independent variables were: blade width (L), cutting depth (L), blade loading distance (L), operating speed (LT⁻¹), and acceleration due to gravity (LT⁻²). The dependent variable was draft force (F). Concept of load-growth curve was used to accurately predict draft requirements of bulldozer blades.</td>
</tr>
<tr>
<td>Freitag et al. (1970a, 1970b, 1977)</td>
<td>Similitude studies of soil-machine system</td>
<td>State-of-the-art review paper related to similitude in soil-machine system. Detailed accounts of relevant soil properties, their measurement, inconsistencies in measured values due to different techniques, desirability of using analog devices to measure soil properties. difficult in scaling soil properties, problems in maintaining same soil properties for model and prototype even when soil preparation techniques were the same, and use of artificial soils were given.</td>
</tr>
<tr>
<td>Sprinkle et al. (1970)</td>
<td>Similitude study with static and dynamic properties of rectangular blades using scale models, use of artificial (Goose lake fire clay and SAE 5W) to scale soil properties was investigated moreover, scaling for speed effect was also found to be important, Inclusion of viscous effects (speed effect improved the prediction at higher speeds.</td>
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<tr>
<td>Authors</td>
<td>Study Type</td>
<td>Artificial Soils</td>
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<tr>
<td>Wang and Liang (1970)</td>
<td>Tillage tool draft</td>
<td>Artificial soils</td>
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<tr>
<td>Luth and Wismer (1971)</td>
<td>Development of prediction equations for soil cutting blades in sand using dimensional analysis</td>
<td>Artificial soils</td>
</tr>
<tr>
<td>Verma and Schafer (1971a)</td>
<td>Distorted models in non-uniform soil profiles</td>
<td>Artificial soils</td>
</tr>
<tr>
<td>Verma and Schafer (1971b)</td>
<td>Distorted models in non-uniform soil profiles</td>
<td>Artificial soils</td>
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<tr>
<td>Author(s)</td>
<td>Title</td>
<td>Description</td>
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<tr>
<td>Schafer et al. (1972, 1977)</td>
<td>Distortion in the similitude studies of soil-machine systems</td>
<td>State-of-the-art review paper that addresses the issue of distortion in similitude studies of soil-machine systems. It lists various sources of distortion and the key role played by soil properties in inducing distortion. Various cases involving purely frictional, purely cohesive, and cohesive-functional soil were considered. Techniques for dealing with distorted models were described in detail.</td>
</tr>
<tr>
<td>Wismer and Luth (1972)</td>
<td>Development of prediction equation; for soil cutting blade; using dimensional analysis</td>
<td>Considered two dependent and nine independent variables to describe soil cutting by blades in almost fully saturated clay. Independent variables were: blade width (L), blade length (L), operating depth (L), blade angle (°), specific weight of soil (FL⁻³), cohesion (FL⁻²), shear rate factor (°), operating speed (LT⁻¹), and acceleration due to gravity (LT⁻²). Dependent variables were: draft force (F) and vertical reaction (F). The soil internal stress of function was negligible, and soil-metal friction was constant. Soil cohesion and shear rate effect were incorporated using a dimensionless term developed from cone index values obtained using a standard cone, blade speed, and blade width.</td>
</tr>
<tr>
<td>Swanson (1973a)</td>
<td>Dimensional analysis to develop prediction equations for performance of dual and tandem rigid wheels in sand</td>
<td>Considered two dependent and seven independent variables in the study of performance of dual and tandem wheels in sand. Independent variables were: wheel diameter (L), wheel width (L), dual wheel spacing (L), frictional soil sinkage parameter (FL⁻²), soil sinkage exponent (-), and load (F). Dependent variables were: motion resistance (F) and wheel sinkage (L). Note that n is the exponent in the plate sinkage equation.</td>
</tr>
<tr>
<td>Swanson (1973b)</td>
<td>Scale model studies of treadless tire tests in clay</td>
<td>Tests with scale model treadless tires were conducted to verify the use of a slightly modified dimensionless clay mobility number (NC) in predicting the performance of treadless pneumatic tires in clay:</td>
</tr>
<tr>
<td>Wismer</td>
<td>Off-road</td>
<td>A set of simple, widely used traction prediction equations for off-road</td>
</tr>
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</table>
| and Luth (1973, 1974) | vehicle traction prediction equations using dimensional analysis | vehicles was developed using three dependent and six independent variables. Independent variables were: tire section width (L), overall tire diameter (L), rolling radius (L), soil cone index value (FL⁻¹), axle load (F), and wheel slip (\(-\)). Dependent variables were: towed force (F), net traction (F), and input torque (FL). The concept of wheel numeric was used to represent soil type and condition.

| Burt et al. (1974b) | Similitude studies related to traction | Considered one dependent and seven independent variables to predict the dynamic net traction of smooth, rigid wheels operating at relatively low speed. Independent variables were: wheel diameter (L), wheel width (L), other pertinent length parameters (L), slip (\(-\)), characteristic soil property (FLₐ), other pertinent soil properties (FL₉ₗ), and dimensionless soil properties (\(-\)). The dependent variable was dynamic net traction (F). The authors did not specify the soil properties but wanted to determine the dimension of the relevant soil property or properties, based on the study of the distortion factor. They found fundamental differences between large prototype and small models that made it impossible to determine the dimensions of the relevant soil property or properties.

| Wismer et al. (1976) and Wismer et al (1976) | Application of similitude to soil-machine systems | Provides a state-of-the-art review of dimensional analysis and similitude, and discusses the application of these principles to pneumatic tires in soft soils, soil cutting, tillage implements, and bulldozer blade.

Traction studies: Considered 13 independent and four dependent variables. Independent variables were: tire diameter (L section height (L), section width (L), deflection (L), cohesion (FL⁻²), internal friction angle (\(-\)), specific weight (FL⁻³), spissitude (FL⁻²T), load (F), wheel...
<table>
<thead>
<tr>
<th>Study</th>
<th>Description</th>
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<tbody>
<tr>
<td>Gee-Clough et al. (1977)</td>
<td><strong>Development of prediction equation for plow draft</strong></td>
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<td>Considered one dependent and seven independent variables to develop a prediction equation for draft of a particular plow working in different soils. The independent variables were: depth of cut (L), width of cut (L), speed (LT⁻¹), acceleration due to gravity (LT⁻²), specific weight (FL⁻³), soil stress (FL⁻²). Three different stress terms were tried: (1) cone index, (2) shear stress ( r = c + p \tan(\Phi) ) where ( p ) is passive earth pressure and ( \mu ) is the angle of internal friction, and (3) ( f = c_s + p\mu ) where ( c_s ) is adhesion and ( \mu ) is the soil-metal friction coefficient. Only the first representation of soil stress (i.e., cone index value) was found to be useful in predicting plow draft.</td>
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<tr>
<td>Pandey and Ojha (1978)</td>
<td><strong>Development of prediction equation; for rigid wheel performance</strong></td>
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<td>The effect of lug height, lug angle, lug spacing, and rim width on the performance of a 685 mm rigid wheel in puddled, lateritic sandy clay loam soil operating at a slow speed was investigated using three dependent and eight independent variables. Independent variables considered were: wheel diameter (L), run width (L), lug height (L), lug...</td>
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<tr>
<td>Author(s)</td>
<td>Method</td>
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<tr>
<td>Wadhwa (1980)</td>
<td>Soil cutting by rectangular blade and angle tool</td>
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<tr>
<td>Ehrlich (1985)</td>
<td>Similitude</td>
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<tr>
<td>Evans et al. (1985)</td>
<td>Similitude of soil-tillage equipment interaction</td>
</tr>
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</table>
| Brixius (1987)     | Traction prediction equation                | Extended the traction prediction equations developed by Wismer and Luth (1973, 1974) by replacing wheel numeric with a more detailed mobility number. The equations were based on three dependent and eight independent variables. Independent variables were: tire section width \( L \), overall tire diameter \( L \), rolling radius \( L \), tire deflection \( L \), section height \( L \), soil cone index value \( FL^{-2} \), axle load \( F \), and wheel slip \( \theta \). Dependent variables were: towed force \( F \), net traction \( F \), and input torque \( FL \). The compressive mobility number \( B_{n} \) used to represent soil was given by:  

\[
B_{n} = \left( \frac{c}{d} \right) \left( \frac{1 + \frac{5b}{h}}{1 + \frac{3b}{d}} \right)
\]

where \( C \) is the cone index value, \( b \) is the section width, \( d \) is the wheel diameter, \( W \) is the axle load, \( S \) is the tire deflection, and \( h \) is the section width. |
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<tr>
<td><strong>Salokhe and Ninh 1993</strong></td>
<td>Used dimensional analysis to develop a soil compaction prediction equation under a pneumatic tire in a clay soil. The authors investigated 11 variables: dry density (ML&lt;sup&gt;3&lt;/sup&gt;) tire section width (L), tire diameter (L), inflation pressure (ML&lt;sup&gt;1&lt;/sup&gt;T&lt;sup&gt;-2&lt;/sup&gt;), cone index(ML&lt;sup&gt;1&lt;/sup&gt;T&lt;sup&gt;-2&lt;/sup&gt;)Moisture content (-), initial dry density (ML&lt;sup&gt;3&lt;/sup&gt;), travel speed (LT&lt;sup&gt;-1&lt;/sup&gt;), axle load (MLT&lt;sup&gt;-2&lt;/sup&gt;), number of passes (-), and gravitational acceleration.</td>
</tr>
<tr>
<td><strong>Canillas and Salokhe (2001, 2002)</strong></td>
<td>Used dimensional analysis to develop a soil compaction prediction equation under a pneumatic tire in three different soils: clay, silty clay loam, and silty loam. The authors investigated eleven variables: bulk density (ML&lt;sup&gt;3&lt;/sup&gt;) and cone index(ML&lt;sup&gt;2&lt;/sup&gt;T&lt;sup&gt;-2&lt;/sup&gt;) as dependent variables, and tire section width (L), tire diameter (L), inflation pressure (ML&lt;sup&gt;1&lt;/sup&gt;T&lt;sup&gt;-2&lt;/sup&gt;), initial cone index (ML&lt;sup&gt;1&lt;/sup&gt;T&lt;sup&gt;-2&lt;/sup&gt;) , moisture content (-), initial dry density (ML&lt;sup&gt;3&lt;/sup&gt;) travel speed (LT&lt;sup&gt;-1&lt;/sup&gt;), axle load (MLT&lt;sup&gt;-2&lt;/sup&gt;), and number of passes(-) as independent variables.</td>
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LESSON 18. Traction

18.1. Introduction

Of the three principal ways of transmitting tractor-engine power into useful work - power takeoff, hydraulic, and drawbar - the least efficient and most used method is the drawbar. Traction is the term applied to the driving force developed by a wheel, track, or other traction device.

18.2. Traction in soils

Soils, like metals, can behave both elastically and plastically. Elastic deformation refers to the ability of the deformed material to return to its original dimensions. Plastic deformation refers to a condition of permanent deformation. For a soil in the elastic condition, a given applied force causes a known deformation. On removal of the force, recovery takes place.

If, however, the force is continually increased, a loading condition will occur that will cause the soil to deform permanently, i.e., it will behave plastically. The onset of this plastic condition is generally considered to be induced by shear failure, i.e., the sliding of one particle over another. In this case, the ability of a particular soil to support a given load before a permanent change to the soil structure occurs is called the shear strength of the soil.

18.3. Shear strength

Granular materials such as soils exhibit cohesive and frictional properties. Cohesion is the bonding together of soil elements irrespective of the type of applied load. Pure clay fits this category; dry sand, on the other hand, exhibits a frictional resistance to shear loads in that the resistance to shear increases with applied load. Most soils exhibit a combination of cohesive and frictional properties. The water content in clay soil has a strong influence on shear strength: the higher the water content, the lower the shear strength. An applied load will result in a wedge-shaped cone below the tire, which will cause deformation and displacement of adjacent soil particles. If the resultant shear stresses are greater then the soil can sustain, sinkage will occur, increasing the surface area. This will continue until the soil is able to support the tire and load.

Two forms of shear strength may be considered:

i. Bulk shear strength: the resistance offered to movement by a relatively large volume of soil aggregates.

ii. Clod shear strength: the resistance offered by the individual clod or aggregate.

The main factors that influence shear strength are:

i. moisture content
ii. packing density and particle size

iii. organic matter content.

Figure 1.1 shows the changes in bulk shear strength with changes in moisture content. The moisture content is also related to the upper and lower plastic limits.

18.4. Plastic limit

As the drying process continues, the plastic state reaches a consistency at which the soil ceases to behave as a plastic and begins to break apart and crumble. The increase in shear strength with decreasing moisture content from the upper to the lower plastic limit is clearly seen. The lower plastic limit represents the maximum moisture content where a farmer can break clods during seedbed preparations without causing structural damage. It is a condition frequently accepted as the upper moisture limit for working soils in agriculture. At high moisture contents, clods are very weak, and susceptible to deformation. The higher the organic matter content, the stronger the aggregates. Aggregates produced on fine sand and silt soils tend to be very weak.

18.5. Coulomb and Micklethwaite equations

The failure of an agricultural soil can be described by Coulomb's equation:

\[ t = c \tan \phi + \sigma \]  \hspace{1cm} (1.1)

where \( t \) is the shear stress of the material, \( c \) the cohesive property of the material, \( \sigma \) the normal stress on the sheared surface, and \( \phi \) the angle of internal shearing resistance of the material.

For saturated clay, the cohesion is independent of the applied normal load, and so

\[ t = c \]  \hspace{1cm} (1.2)

Sandy soils, however, have little cohesion but have larger values for the angle of internal shearing resistance. Thus for a sandy soil,

\[ t = \sigma \tan \phi \]  \hspace{1cm} (1.3)

An agricultural soil is composed of both sand and clays, and therefore has properties intermediate between those for sand and clay alone. The typical agricultural soil has therefore both cohesive and frictional properties. Figures 1.2 and 1.3 show the shear versus normal stress characteristics for clay and sand soils, respectively.

Micklethwaite (1944), as cited by Reece (1966), proposed the following modification to the Coulomb equation.

Let \( A \) be the contact area; then

\[ t = \frac{W}{A} \tan \phi \]  \hspace{1cm} (1.4)

If \( t = t_{\text{max}} \) then \( t_a \) is the maximum thrust, or \( H_{\text{max}} \).

The normal stress \( s = \frac{W}{A} \), and so
Mechanics of Tillage and Traction

This expression is usually referred to as the Micklethwaite equation. The pressure under a rigid wheel on frictionless soils at small sinkages was given by Reece (1966) in citing Uffelmann (1961) as

\[ p/c = 5.7 \]  

(1.6)

where \( p \) is the pressure. The sinkage \( z \) necessary to support the wheel for radial pressures given by the above equation is

\[ z = W^2/(5.7c)^2b^2d \]  

(1.7)

Where,

- \( W \) - vertical load on the wheel,
- \( b \) - width of the wheel, and
- \( d \) - wheel diameter.

The rolling resistance can be considered as principally due to the work done in forming the wheel rut.

The distance moved is the sinkage \( z \), and the work done is given by

\[ E_R = 5.7cbz \]

where \( E_R \) is the energy required to form the rut. Thus the rolling resistance \( R \) (assuming that it is due entirely to rut formation) is given by

\[ R = W^2/5.7cbd \]

For a driven wheel, the maximum (drawbar) pull can be obtained by considering the thrust that can be developed given a value for the maximum shearing stress along the contact patch:

\[ F_D = H - R \]

Thus, for frictionless soils that have a maximum shear stress equal to \( c \), the drawbar pull is

\[ F_D = cbr \sin - W^2/5.7cbd \]

where \( \theta \) is the angle of sinkage given in figure 1.4.

**BEKKER THEORY**

Fundamental to the Bekker approach to the theory of land traffic is the relationship between the sinkage \( z \) and the normal pressure \( p \). The relationship developed by Bekker is a modification of an assumed linear relationship between pressure and sinkage that is used in civil engineering soil mechanics for small sinkages. Bekker (1960) gives the following equation:

\[ H_{\text{max}} = cA + W \tan f \]  

(1.5)

where,
Mechanics of Tillage and Traction

$k_c$ - the cohesive modulus of sinkage,

$k_f$ - frictional modulus of sinkage, and

$n$ - exponent reflecting the hyperbolic shape of the load sinkage curve.

The values of $k_c$, $k_f$, and $n$ can be determined for any given soil by conducting load sinkage studies on two plates with different areas. Log-log plots of pressure against sinkage will give straight-line relationships of slope $n$. Two equations for $p$ at $z = 1$ enable values for $k_c$ and $k_f$ to be obtained.

The horizontal shear stress is given by a modification of the Coulomb-Micklethwaite equation. For plastic soils, the following relationship is given by Bekker (1969):

$$\text{---------} (1.9)$$

Where,

$K$ - slip coefficient and

$j$ - the amount of soil deformation that produces stress $t$

The Coulomb constants $c$ and $\phi$ can be determined for a given soil by plotting maximum shear stress against normal pressure to give the straight-line equation. The slip coefficient (also termed the deformation constant) can be obtained from stress-deformation curves that are obtained with a bevameter (A bevameter is a device used to determine, in situ, the pressure-sinkage relationship for a given supporting surface. It is usually mounted to a vehicle subframe, which acts to provide force reaction during the penetration test).

Figure 1.5 shows shear stress-deformation curves for plastic and brittle soils (Bekker, 1969). If an annular shear ring is used, then the shear strength is measured in terms of the shear torque, and the displacement is measured in terms of the angular deformation. These tests are usually repeated at various levels of normal pressures. Idealized shearing stress-deformation graphs for different normal pressures are shown in figure 1.6. The yield points I, II, III, IV mark the end of the quasi-elastic deformation and the beginning of plastic flow. This yield point, for a given soil, will occur at the same deformation value and represents the constant $K$.

The idealized shear stress-deformation curves may not be obtained in practice. On soft ground, and at high loads that are greater than the bearing capacity of the soil, the shear stress may continue to rise with deformation without giving any exact yield point. This situation requires correction of the previous equation for $\phi$.

Slip $i$ can be expressed in terms of the deformation $j$ and the distance $x$ measured from the start of the ground contact area and some location along the ground contact area:

$$j = ix$$

The thrust $H$ at a particular slip can be found by using the above relationship and the equation for horizontal shear stress. Integration over the track length $l$ gives

$$\text{---------} (1.10)$$

where $H_i$ is the thrust at slip $i$. 

---

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The rolling resistance $R_1$ is determined by considering the work done in making a rut of length $l$ and of depth $z$:

\[
R_1 = \frac{WL}{l} = \frac{W}{z}
\]  \hspace{1cm} (1.11)

Substituting for the radial pressure $p$ gives

\[
R_1 = \frac{p}{z}
\]  \hspace{1cm} (1.12)

For a wheel, Bekker provides mathematical approximations for the sinkage $z$ and the rolling resistance $R$, such that

\[
R = \frac{D}{z} - \frac{W}{z}
\]  \hspace{1cm} (1.13)

where $D$ is the wheel diameter, and

\[
D = \frac{W}{R} = \frac{W}{z}
\]  \hspace{1cm} (1.14)

The drawbar pull $f_d$ can then be determined from

\[
F_D = H_i - R_{\text{wheel}}
\]  \hspace{1cm} (1.15)

A slightly modified version of the equation for the tractive force developed by a tire was given by Bekker (1956) in terms of the slip $i$, where

\[
f_d = \frac{K}{l} - \frac{W}{l}
\]  \hspace{1cm} (1.16)

and

\[
f_d = \frac{K}{l} - \frac{W}{l}
\]  \hspace{1cm} (1.17)

where $l$ is the length of the contact area ($= 2[\gamma(d-\gamma)]^{\gamma}$), $\gamma$ the tire deflection, $d$ the tire diameter, and $K$ the tangent modulus of deformation from the ring shear test.
LESSON 19. Traction mechanics

19.1. Traction Mechanics

Fig. 2.1(a) illustrates one of several alternative methods of describing the forces acting on a wheel. The figure is divided into three distinct force states: braked, driven, and driving. The transition point between the braked and driven force states is the towed wheel condition. A towed wheel is unpowered: axle torque is zero neglecting bearing friction. The transition point between the driven and driving force states is the self-propelled wheel condition. For a self-propelled wheel, pull is zero with the applied torque simply overcoming the motion resistance of the wheel.
Mechanics of Tillage and Traction

The curves presented in Fig. 2.1(a) represent a given soil strength, tire size, and load. As soil strength increases, the curves move upward to the left, as soil strength decreases, they move downward to the right.

In Fig. 2.1(a), both the axle torque and pull are plotted as functions of wheel slip. These reactions develop from soil stresses resulting from slippage (motion loss) of the wheel. Slip is defined as

\[
S = 1 - \frac{\dot{V}_a}{V_t}
\]

Where,

\( S \) - wheel slip

\( \dot{V}_a \) - actual travel speed

\( V_t \) - theoretical wheel speed = \( rw \)

\( r \) - rolling radius of wheel on hard surface

\( w \) - angular velocity of wheel

The term "rolling radius" is defined in ASAE S 296.2 (Agricultural Engineers Yearbook 1978) as 'the distance traveled per revolution of the traction device divided by 2\( \pi \) when operating at the specified zero condition. The zero condition selected here is the vehicle operating in a self-propelled condition on a hard surface, such as a smooth road, with zero drawbar load. This differs from another common zero condition, which is the self-propelled, zero drawbar load condition on the test surface.

The towed force point and driving wheel states are particularly important. Free body diagrams of the wheel for these two conditions are shown in Figs. 2.1(b) and (c).

In Fig. 2.1(b) for the towed wheel, the soil reaction \( G \) is resolved into a horizontal component (which from equilibrium considerations must be equal and opposite to the towed force \( TF \) exerted at the axle center) and a vertical component \( R \) (which must be equal and opposite to the wheel load \( W \)). The horizontal component of the soil reaction is assumed to act at a distance \( r \) below the wheel center. Note that \( W \) includes both the weight of the wheel and any vertical reaction force from the vehicle on which the wheel is mounted. This vertical reaction force will be affected by weight transfer.

Since there is no axle torque acting on the towed wheel,

\[
TFr - Re = 0 \quad \text{----------------------------------- (2.2)}
\]

or \( e = (TF/R)r = (TF/W)r \). Since \( \rho = TF/W \) has been defined as the motion resistance ratio, \( e = rr \).

In Fig. 2.1(c) for the driving wheel, the soil reaction \( G \) is again resolved into horizontal and vertical components. However, the horizontal component is again assumed to act at a distance \( r \) below the wheel center and is now divided into two forces; a gross traction force \( F \) and a motion resistance force \( TF \). As the symbol implies, the motion resistance force acting on the driving wheel is considered to be the same as the towed force for the wheel.

Summing forces in the horizontal direction
Mechanics of Tillage and Traction

\[ H = F - TF \]  

- \hline

Defining \( \mu_g = F/R = F/W \) as the gross traction coefficient and that \( \mu = H/R = H/W \) was defined as the net traction coefficient, dividing equation 6.20 by \( W \) gives the relation:

\[ \frac{H}{W} = \mu = \frac{F}{W} - \frac{TF}{W} = \mu_g - \rho \]  

- \hline

Summing the moments acting on the wheel,

\[ T - (F - TF)r - Re = 0 \]  

- \hline

By using equation 2.2,

\[ T = Fr \]  

- \hline

Thus the wheel torque \( T \) is assumed equal to the gross tractive force \( F \) acting at a moment arm equal to the rolling radius \( r \).
LESSON 20. Off road traction

20.1. Introduction

The primary purpose of agricultural tractors, especially those in the middle to high power ranges, is to perform drawbar work. The value of a tractor is measured by the amount of work accomplished relative to the cost incurred in getting the work done. Drawbar work is defined by pull and travel speed. Therefore, the ideal tractor converts all the energy from the fuel into useful work at the drawbar. In practice, most of the potential energy is lost in the conversion of chemical energy to mechanical energy, along with losses from the engine through the drivetrain and finally through the tractive device. Research shows that about 20% to 55% of the available tractor energy is wasted at the tractive device/soil interface. This energy wears the tires and compacts the soil to a degree that may cause detrimental crop production (Burt et al., 1982).

Efficient operation of farm tractors includes:

(1) maximizing the fuel efficiency of the engine and drive train,

(2) maximizing the tractive advantage of the traction devices, and

(3) selecting an optimum travel speed for a given tractor-implement system.

Throughout the years, official tractor performance drawbar tests have been conducted on hard surfaces and in recent years (30+ years) on concrete. While this provides a valid comparison between tractors, the data does not provide much information about performance under field conditions. The primary difference between official tests and field conditions is the performance of the tires or other tractive devices.

20.2. Solid Wheel on a Hard Surface

An understanding of traction mechanics is fundamental to understanding differences between tractive performance and tractor performance. The basic forces involved in a powered wheel are shown in figure 1 for the simple case of a solid wheel on a hard surface. The torque input (T) develops a gross traction (GT) acting at the contact surface. Part of the gross traction is required to overcome motion resistance (MR), which is the resistance to the motion of the wheel, including internal and external forces. The remainder is equal to the net traction (NT) that the wheel develops, given by NT = GT - MR.
Mechanics of Tillage and Traction

Fig. 3.1. Basic wheel forces for a solid wheel on a hard surface

W = Weight, static
Wd = Weight, dynamic
sir = Loaded radius, static
rt = Rolling radius
T = Torque radius
Vt = Velocity, theoretical
Va = Velocity, actual
T = Axle torque
GT = Gross traction (theoretical pull)
NT = Net traction (actual pull)
MR = Motion resistance

*****😊*****
Lesson 21. Traction Model

21.1. Introduction

Of the three principal ways of transmitting tractor-engine power into useful work—power takeoff, hydraulic, and drawbar—the least efficient and most used method is the drawbar.

Traction is the term applied to the driving force developed by a wheel, track, or other traction device.

Tractive efficiency (TE) is defined as the ratio of output power to the input power for a traction device. It is the measure of the efficiency with which the traction device transforms the torque acting on the axle into linear drawbar pull. Several factors lower the tractive efficiency; among these are steering, rolling resistance, slip, and friction in, and deflection of the traction device.

Net traction coefficient (k) is defined as the ratio of the net pull produced to the dynamic normal load on the traction device. The difference between traction efficiency and coefficient of net traction should be recognized.

Motion resistance ratio (k) is defined as the rolling resistance force divided by the normal load on the traction device.

21.2. Traction Model

If a plate of width b and length l is equipped with lugs sufficiently long, such that an area A = bl shears off as in Fig. 1.1a, we find that the force required is usually dependent upon both the normal force and the area.

If we plot maximum values of F against W (as in Fig. 1.2), we find for soils having some cohesion that F does not approach zero as W approaches zero. If the maximum values are plotted for a soil that has both cohesion c and internal friction, the result will be similar to Fig. 1.2. The equation for such a curve is

\[ F = Ac + W \tan\phi \]  \hspace{1cm} (1.1)

or

\[ F = A(c + p \tan\phi) \]  \hspace{1cm} (1.2)

where, \( p = \frac{W}{A} \) is the average normal soil pressure and A is the area.

For a track

\[ p = \frac{W}{bl} \]  \hspace{1cm} (1.3)
where, \( b \) is the width of the track and \( l \) is the length of the track in contact with the soil. Uniform pressure is assumed.

For a rubber tire, the "footprint" is approximately in the shape of an ellipse (Fig. 1.3) for which case, \[ p = \frac{W}{0.78bl} \] (1.4)

If we know the soil values \( c \) and \( \phi \), the maximum soil thrust can now be approximated by equation 1.2.

This simple model is not very useful because we seldom know \( c \) and, \( p \) is not uniform. In addition the model does not predict the magnitude of the motion resistance force acting on the traction device.

Fig 1.1. Method of determining maximum values of shearing force for various levels of vertical loading

Fig 1.2. The soil parameters \( c \) and \( \phi \) can be determined from a plot of maximum values of the shearing force \( F \) versus the normal force \( W \)
Fig 1.3. Soil thrust from a traction member affected by both shear area and weight

21.3. WEIGHT TRANSFER

The tractive ability is affected by the vertical soil reaction against the traction wheels. Weight transfer, from drawbar pull, decreases the soil reaction against the front wheels by an amount and increases the reaction against the rear wheels by an amount $\Delta R_f$, thus adding to the maximum drawbar pull for a two-wheel-drive tractor. The symbols in the "weight transfer" equations are defined in Fig. 1.4.

\[
\Delta R_f = \frac{p y_f}{L_1} \quad \text{(1.5)}
\]

\[
\Delta R_r = \frac{p y_r}{L_1} \quad \text{(1.6)}
\]

Any means of increasing the rear wheel reaction will increase the traction of the rear wheel if the soil has sufficient strength and if sinkage does not limit the traction.

21.4. FOUR-WHEEL-DRIVE TRACTORS

The path or wheel track produced by the front wheels of a 4WD tractor having equal-sized tires usually increases the soil strength for the rear tires. The stronger soil increases the traction and also decreases the rolling resistance for the rear wheels. As a result, both the net tractive coefficient and the tractive
efficiency should be greater for a 4WD tractor than a 2WD tractor when operating on soft compactible soil.

Reed et. al. (1959) found (Fig. 1.5) that at 10% slip on Hiwassee sandy loam soil, the tractive efficiency of the 2WD was 56%, while that of the tandem drive was 66%.

![Fig 1.5. Power efficiency and travel plan reductions for two-wheel and tandem four-wheel-drive tractors. Data are for tractors of equal weight with the dynamic load equally distributed on all wheels for the tandem drive. (from I.F.Reed, A.W.Cooper, and c.A.Reeves, Trans. of ASAE, Vol. 2. No.1, 1959)](image)

Dwyer and Pearson (1975) also found that the traction of the rear wheels (second pass) was greater, especially in soft soil. Dwyer found that the rolling resistance of the second pass was less than on the first pass. The average maximum drawbar power of a 4WD tractor with equal-sized wheels in all field conditions was 14% greater than that of a 2WD tractor. A 4WD tractor with unequal-sized wheels had 7% greater drawbar power than did the 2WD tractor.

### 21.5. TREAD DESIGN

The design of the tread on tires has been the subject of much debate because the performance of one tread compared to other treads is affected by the traction conditions and the performance criteria being used. Some of the available tire treads are illustrated in Figs. 1.6 and 1.7.
Mechanics of Tillage and Traction

purpose tread; (b) R-2 or deep lug cane and rice tread; (c) R-3 or flotation; and (d) R-4 or industrial lug.

Fig. 1.7. (a) European-style tire with wider lugs for improved road wear. (Courtesy The Goodyear Tire and Rubber Co.) (b) Japanese style tire for rice soil conditions.

21.6. TRACTION CONDITIONS AFFECTING PERFORMANCE

Some traction conditions that affect performance are:

1. Soil parameters (physical properties)
2. Presence of crop residues and cover crops
3. Direction of loading of tire (e.g., hillside use)
4. Load carried by tire
5. Tire pressure (deflection ratio = deflection/tire section height)

The decision as to which tire tread performs best is also dependent on the criteria being used. Some of the criteria are:

1. Tractive efficiency
2. Net tractive coefficient
3. Tire life
4. Soil compaction
LESSON 22. TRACTION IMPROVEMENT AND TRACTION PREDICTION

22.1. TRACKS

Tracks have been used for many years to reduce soil pressure and to increase traction on soft, loose soil surfaces that have low strength. Tracks result in the best relative performance as compared with pneumatic tires when the tractor is operating at nearly maximum drawbar pull on soft, loose soil surfaces.

22.2. TRACTION IMPROVEMENT

For certain soil conditions, traction aids are helpful. Table 1.1 shows the relative improvement of three traction aids. Strakes and halftracks are more commonly used in Europe. Table 1.2 shows the relative effect of adding weight and increasing the contact area (larger tires). Tractors with both rubber tires and wheel extensions (strakes) are commonly used on weak surfaces such as rice paddy soils.

Rubber traction tires, as compared to steel traction wheels, have greatly improved the tractive efficiency, the maneuverability, and the comfort of farm tractors. Except on very firm soils, however, rubber tires have not increased the traction. In fact, under some conditions, such as when the surface of the soil is very wet and slick or when the soil is covered with a thick cover crop, the traction of rubber tractor tires is poor.

Table 2.1. Relative Improvement in Traction from Chains, Strakes, and Half-tracks on 13.6-28 (345-711 mm) Tires

<table>
<thead>
<tr>
<th>Traction Aid</th>
<th>Pull at 15% Slip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sandy Loam: Cultivated, Loose, and Dry</td>
</tr>
<tr>
<td>Air-filled tires only</td>
<td>1.00</td>
</tr>
<tr>
<td>Air-filled tires plus:</td>
<td></td>
</tr>
</tbody>
</table>
### Table 2.2. Relative improvement in traction performance of larger, dual and solution – filled tires

<table>
<thead>
<tr>
<th>Tire and Arrangement</th>
<th>Pull at 15% Slip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sandy Loam: Cultivated, Loose, and Dry</td>
</tr>
<tr>
<td>Air filled (13.6-28)</td>
<td>1.00</td>
</tr>
<tr>
<td>Air-filled dual (13.6-28)</td>
<td>1.05</td>
</tr>
<tr>
<td>Large air filled (14.9-28)</td>
<td>0.97</td>
</tr>
<tr>
<td>Solution filled (13.6-28)</td>
<td>1.61</td>
</tr>
<tr>
<td>Solution-filled dual (13.6-28)</td>
<td>2.36</td>
</tr>
</tbody>
</table>
When traction conditions are good, the largest improvement in traction can be made by simply adding more weight to the tractor drive wheel. However, when the surface soil is very wet the internal friction, , approaches zero, and therefore an increase in the soil pressure will not increase traction significantly unless the traction device can "cut through" the low friction surface layer.

22.3. TRACTION PREDICTION FROM DIMENSIONAL ANALYSIS

22.3.1. EQUATION DEVELOPMENT

Dimensional analysis is used to simplify the prediction equations for the multivariable system. The variables considered in the following prediction equations are presented in Table 2.3.

As shown, nine pertinent variables are involved in the traction equations. Seven dimensionless ratios are needed to formulate a prediction equation. An adequate set of dimensionless ratios relating the variables is

\[
\rho \left( = \frac{TF}{W} \right), \mu \left( = \frac{H}{W} \right), \mu_0 \left( = \frac{F}{W} \right), = f \left( \frac{C_I b d}{W}, \frac{b}{d}, \frac{r}{a}, S \right)
\]  \hspace{1cm} \text{(2.1)}

However, since \( \mu = \mu_g - \rho \), experimental relations only need be developed for \( \rho \) and \( \mu_g \).

Table 2.3. Wheel soil model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cone index</td>
<td>CI</td>
<td>FL^2</td>
</tr>
<tr>
<td>Wheel:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tire section width</td>
<td>b</td>
<td>L</td>
</tr>
<tr>
<td>Overall tire diameter</td>
<td>d</td>
<td>L</td>
</tr>
<tr>
<td>Tire rolling radius</td>
<td>r</td>
<td>L</td>
</tr>
<tr>
<td>System:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td>W</td>
<td>F</td>
</tr>
<tr>
<td>Towed force</td>
<td>TF</td>
<td>F</td>
</tr>
<tr>
<td>Pull</td>
<td>H</td>
<td>F</td>
</tr>
<tr>
<td>Gross tractive force</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Slip</td>
<td>S</td>
<td>–</td>
</tr>
</tbody>
</table>

i. Towed Force: The towed force or motion resistance of a pneumatic tire is dependent on load, size, and inflation pressure, as well as soil strength. For soils that are not very soft and tires that are operated at nominal tire inflation pressures, the towed force can be predicted from

\[
\rho = \frac{TF}{W} = \frac{1.2}{C_n} + 0.04
\]  \hspace{1cm} \text{--------------------------------- (2.2)}
Mechanics of Tillage and Traction

\[ C_n = \text{wheel numeric} = \frac{CIbd}{W} \quad \text{(2.3)} \]

(Dimensions must be selected such that the wheel numeric is dimensionless.)

CI = cone index measured with a cone penetrometer as in ASAE R 313.1

It should be noted that for a firm surface such as compacted dry clay, the \( C_n \) value would be very large and the towed force would be equal to 0.04 times the wheel load. This rolling resistance is attributed to tire flexing and scrubbing. Equation 2.3 was developed for tires with a tire width/diameter, b/d ratio of approximately 0.3. Any large deviation from this width/diameter ratio can be expected to change the quantitative relation of the towed force function. The towed force number defined by equation 2.3 is presented graphically in Fig. 2.1.

\[ \text{Fig. 2.1. Towed wheel performance relation (Wismer and Luth, 1974)} \]

ii. Gross Tractive Force: The variations of the gross tractive force with soil strength and slip have been incorporated into a relation including the effect of wheel load and tire size:

\[ \mu_g = \frac{F}{W} = \frac{T}{W} = 0.75 \left( 1 - e^{-0.3C_nS} \right) \quad \text{------------------ (2.4)} \]

Where, e = base of natural logarithms.

iii. Pull: Substituting equations 2.3 and 2.4 into equation 2.4 in module VI, the net traction coefficient \( \mu \) is given by

\[ \mu = \frac{H}{W} = 0.75 \left( 1 - e^{-0.3C_nS} \right) - \left( \frac{1.2}{C_n} + 0.04 \right) \quad \text{-------------- (2.5)} \]

A practical restriction of b/d 0.30 is imposed on the final equation along with a tire deflection/section height ratio (\( /h \)) limitation of 0.20. The restriction on d/h is associated with an r/d 0.475. This reduces the pull relation to one dependent (H/W) and two independent (\( C_n \) and S) dimensionless ratios
resulting in equation 2.5, which is presented graphically in figure 2.2. Prediction equations 2.2, 2.4 and 2.5 give good estimates of the performance of a single tire except where the surface of the soil is weak. However, the effect of tread differences cannot be predicted with the equations.

iv. Tractive Efficiency: The pull, torque and slip characteristics of a driving wheel define both the magnitude and efficiency of tractive performance. The pull/weight ratio or net tractive coefficient is an accepted term for defining performance level. Similarly, the term tractive efficiency (TE) has been adopted to define efficiency. Tractive efficiency of a wheel is defined as:

\[
TE = \frac{\text{output power}}{\text{input power}}
\]  

(2.6)

Which can be expressed as,

\[
TE = \frac{HV_a}{T \omega} = \frac{HV_a}{T \left( \frac{V_t}{\gamma} \right)} = \frac{\left( \frac{H}{W} \right)}{\left( \frac{F}{W} \right)} \left( 1 - S \right)
\]  

(2.7)

Fig. 2.2. Driving wheel prediction relation (Wismer and Luth, 1974)

The variation of tractive efficiency and the pull/weight (H/W) ratio of a driving wheel with slip is shown in figure 2.3. It is readily observable that TE reaches a maximum at a relatively low slip and then decreases with increasing slip. Also note that the maximum TE occurs at lower slip values for the large \( C_n \) values that are associated with higher soil strengths or lower wheel loadings. Maximum power output of a wheel occurs at the wheel slip of maximum TE. However, the H/W ratio is not close to its maximum value at this slip. The requirement for a large drawbar pull necessitates that the design slip be selected to the right of the slip corresponding to the peak of the TE curve. A typical design TE curve is shown in figure 2.3. From this curve the design TE, H/W, and slip for a variety of soil strengths and wheel loading combinations, in terms of \( C_n \), can be determined: for example, for \( C_n = 30 \), TE = 0.72,
H/W = 0.51, slip = 0.16. This approach permits balancing the design of the vehicle over the range of soil strengths it will probably encounter in its operational life.

Fig. 2.3. Tractive performance of wheels on soil (Wismer and Luth, 1974)
LESSON 23. CONE INDEX AND TIRE BASICS

Cone Index: Cone index is used as the measure of soil strength in the traction equations. Cone index is the average force per unit base area required to force a cone-shaped probe into soil at a steady rate. The design and use of the cone penetrometer is discussed in ASAE R 313.1. (Agricultural Engineers Yearbook, 1977).

Cone index characteristically varies with depth of penetration (figure 2.4). Thus the question arises as to what cone index value should be used. For the traction equations, the 0 to 6 in. (15 cm) average cone index has produced the best correlations for machines with tire sinkages of less than 3 in. (7.5 cm). However, if the tire sinkage is greater than this value the cone index should be averaged over the 6-in. (15 cm) layer, which includes the maximum sinkage of the tire. In general, cone index should be measured before the soil is subjected to wheel traffic.

Highly compactible soils, such as freshly tilled soils, present a special problem in predicting tractive performance. The soil tends to compact and increase in strength under heavy tire loads. Cone index measured after traffic may be several times the value measured before traffic. Best results to date have been accomplished by using after-traffic cone index values in the developed equations for highly compactible soils. No satisfactory method has been devised for predicting after-traffic cone index from before-traffic measurements.
23.1 Tire Size, Load, and Air Pressure Relationship

The tire companies, through the RMA (Rubber Manufacturers Association), have determined load and torque limits for each tire. This information (see Appendix) is published in the form of standards by SAE (J709d) and by ASAE (S295.1). Using this information, it is possible to select the minimum size tire for a given tractor load condition. Such a tire will be correctly sized for the vertical load and torque. However, it may be too small for the soil conditions. Ellis (1977) has simplified the problem by using the tractor power and operating speed as a basis for selecting the proper tire. From this graph (see Fig. 11.13), one can also select the proper combination of dual tires. Note that the use of dual tires does not double the power that can be transmitted by the tires.

Because a tractor may be used in a variety of soil conditions and loads, the manufacturer will have several sizes, treads, and ply ratings available for each tractor. One manufacturer of a popular 2WD tractor with 105 kW advertises 10 different tires available plus 5 different dual arrangements for the rear driving wheels. The same tractor also has 6 different front tire sizes available.

23.2 Radial-Ply Construction

The advantage of using radial-ply tractor tires as compared to bias-ply tires is not as pronounced for tractors as for highway vehicles where the increase in life (mileage) and the decrease in fuel consumption justify the extra cost. Radial-ply tires have been used to a greater degree on tractors in Europe than in North America, possibly because European tractors are used more for highway transportation. In addition, the cost of fuel is much greater.

The advantage of the radial-ply tractor tire in significantly improving the coefficient of traction under most all conditions is shown in Fig. 11.14 from a study by Dwyer (1975). Taylor et al. (1976), at the
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National Tillage Machinery Laboratory, found a smaller advantage for the radial-ply tire. Taylor found that on five of seven soils, the coefficient of net traction at 15% slip increased 6% to 18%. However, the tractive efficiency of the radial-ply tire was only slightly higher. On soft soils, there was no advantage for the radial-ply tire.

23.3 Effect of Lug Spacing

A study was made by Taylor (1974) on the effect of lug spacing on 11.0-38 (279-965 mm) tires. The number of lugs per side on the five tires tested was 20, 23, 26, 29, and 32, which gave a pitch (in millimeters) of 238, 207, 178, 164, and 148. Taylor found (see Fig. 11.15) that when the tires were tested on sod the maximum pull occurred when using the 23-lug tire. On the other soil conditions tested, lug spacing had little effect.

****itical Figure 11.14 Comparison of coefficients of traction at 20% slip (upper curves) and rolling resistance (lower curves) for radial (O) and cross-ply (X) tires. (From Dwyer, 1975.)****
LESSON 24. TIRES FOR AGRICULTURAL TRACTORS

24.1. TIRES

A tire forms a torus of a complex flexible reinforced composite material surrounding a steel hub and filled up with air under pressure. It has more or less expanded ribs on the rolling tread. The different types of torus shapes and construction are selected according to the tire application. In general, a tire is characterized by the tire construction type, (figure 3.1), mounting data, ply rating and other dimensions such as that shown in figure 3.2.

![Tire construction type](image)

Fig. 3.1. Tire construction type
24.2. TIRE DEVELOPMENT

The steel-wheeled tractor was replaced by vehicles fitted with pneumatic tires in the 1920s and 1930s. Harvey Firestone (1868-1938) was instrumental in developing reinforced, "low-pressure" tires that were suitable for agricultural field work. World War II resulted in shortages of natural rubber previously used for tire production. This accelerated the research and development of synthetic rubbers such as styrene and latex. The synthetic rubbers produced had better wear-resistant properties than the original rubber tires. Synthetic rubbers that are commonly used today include polyurethane, neoprene, polybutadiene, and butyl. Natural rubber may still be used for some specialized applications.

24.3. TIRE FUNCTION

The agricultural tire must perform the following functions:

1. support the vehicle and associated loads at some low level of ground pressure,
2. absorb shock loads and cushion the vehicle against minor surface irregularities,
3. provide traction (and braking),
4. provide for steering and directional stability,
5. resist the abrasive action of the various surfaces on which it is expected to operate.
24.4. CATEGORIES OF TIRE APPLICATIONS

The type of application (of tyres as traction elements) can be divided into the following four broad categories, each of which is comparatively unique.

1. In the construction field, as typified dam, waterway and highway projects which require movement of large quantities of earth and rock. In this type of service, speeds as high as 40 to 80 km h\(^{-1}\), length of haul to 16 km, and size of loads and equipment to 75 m\(^3\) capacity are generally expected.

2. In the logging, mining and petroleum industries – heavier units such as mobile cranes and self contained pumps and power plants, are used. This demands tire types with high floatation characteristics and load carrying capacities.

3. In military operations. In this field the various types of tired vehicles are expected to operate over a great variety of surfaces in cross-country transport. Reliability is of particular importance.

4. In general transportation into newly developed areas without adequate highways or railways – such vehicles must have floatation and mobility capability under heavy loads, without the need for extensive preparation and maintenance of roads or tracks.

24.5. TIRE SIZES AND TYPES

The growth of off-road operations has brought about a great diversification in tires to meet all service requirements.

1. Number of sizes. Tires have become larger both in cross-section and in rim diameter. Larger tires permit higher loads per tire without sacrificing floatation.

2. Conventional vs wide base. Two types of tires now exist namely, conventional and wide base. Figure 3.3 illustrates the differences between these two types of tires. Without changing the rim diameter or tire overall diameter, the cross-section width can be increased by using a wider rim. With the same tire loading, inflation pressures on the wide base tire can be reduced. The wider cross-section gives improved traction and floatation and the lower unit ground pressure can improve the resistance to damage from stones and other objects.

3. Low section-height tires. The wide base principle can be extended into low section-height tires. Figure 3.4 compares two tire types. The low section-height shape makes possible a wider cross-section for improved floatation without increasing overall diameter or tire weight, as would have been necessary if conventional tire shapes had been maintained.

4. Single vs dual. Another application of low-section height principle is in the use of larger simple tires to replace dual tires (figure 3.5). Although the change from dual tires to on large single tire reduces total ground contact area, experience has shown that floatation and mobility are improved without reduction in total oad carrying capacity.

5. Tread pattern. All off highway operations do not need the same degree of traction. As a result, separate tread designs are used for different degrees of tractive effort.
24.6. TREAD

The important variables in relation to the tyres include:

(i) size (diameter and width) which determines their tractive capacity and rolling resistance.

(ii) strength, expressed in terms of ply rating, which in turn determines the pressure that can be used and hence the weight that the tire can carry; this in turn also determines the tractive capacity and the rolling resistance.
(iii) tread pattern which, together with the surface characteristics, determines the engagement and / or contact with the surface.

The losses in power at the wheel / surface interface are often great, particularly on soft surfaces (ie, their efficiency is low), hence the power available at the tractor drawbar may be much less than the power of the engine. Hence the choice of the tires and the weight on them is crucial in determining the overall performance of the tractor.

Various types of wheels and / or tyres may be used on the tractor, depending mainly on the surface on which it is working. For the following conditions, the tyres or wheels indicated are recommended as shown in Figure 3.6.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Tread form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Hard surfaces such as roads</td>
<td>Large area, shallow tread with ‘high’ pressure</td>
</tr>
<tr>
<td>(b) Normal agricultural work, dry soil</td>
<td>Heavy, intermediate depth tread</td>
</tr>
<tr>
<td>(c) Soft, wet agricultural soils</td>
<td>Deep tread</td>
</tr>
<tr>
<td>(d) Lawns, low sinkage is required</td>
<td>Wide, low pressure</td>
</tr>
<tr>
<td>(e) Dry soil, heavy loads as in earth</td>
<td>Tracks, as on a “crawler” tractor</td>
</tr>
<tr>
<td>moving</td>
<td>Saturated puddle soils</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------</td>
</tr>
</tbody>
</table>

*****😊*****
LESSON 25. TIRE TERMINOLOGY AND SELECTION OF TIRES

25.1. TIRE TERMINOLOGY

The prefix R has been adopted by ASAE (1983) as the code for rear, or drive, tires. The prefixes F and I have been adopted for steering tires (or front tires) and for implement tires, respectively. The "regular" agricultural drive tire is coded R-1. There are three other categories of drive tire: cane and rice (R-2), industrial and sand (R-3), and industrial tractors (R-4). Front or steering tires are coded F-1 for single-rib tires, F-2 for "regular" agricultural tires, and F-3 for industrial multiple-rib tires. Implement tires are coded 1-1 for ribbed, 1-2 for moderate traction, 1-3 for traction, and 1-6 for smooth implement tires.

Tire sizing is based on the tire section width and the rim diameter. Thus a 18.4-26 drive tires has a section width of 18.4 in. and a rim diameter of 26 in. Early tire designs had tire section widths equal to the tire section height. The aspect ratio of these tires, as defined by \( h/b \), where \( h \) is the tire section height and \( b \) the tire section width, was thus equal to 1.0. The length of the contact patch is given by

\[
l \approx 0.31d \quad \text{-------------------------------------------- (4.1)}
\]

where \( l \) is the length of the contact patch and \( d \) the diameter of the unloaded tire. The contact patch can be regarded as elliptical in shape, and the cross-sectional area of the patch is given by

\[
A = \pi lb \quad \text{-------------------------------------------- (4.2)}
\]

where \( b \) is the width of the contact patch.

Today's tires have wider section widths, and aspect ratios of 0.85 or even 0.75 are found (Inns and Kilgour, 1978).

The ply rating of a tire is used to indicate its load-carrying ability. The ply rating was once used to specify the number of plies built into the construction of the tire. This is no longer the case, but the term has been retained as an index of the tire strength. There are three types of casing construction: cross ply, radial ply, and belted bias (Anslow and Warrilow, 1970). In the case of cross-ply construction, the cords are arranged at an angle of approximately 40° to the circumferential centerline of the tire. This is referred to as the crown angle (Wong, 1978). There are normally two or more layers or cords, or plies, each layer being set in the opposite bias. The majority of agricultural tires are of this construction.
The advantage of this construction for off-road vehicles is that the sidewalls have reinforcement, thus providing some protection against their impact damage. Fly tires are constructed using individual cords running from bead to bead in a line perpendicular to the circumferential centerline of the tire. Thus the crown angle is 90°. There are also a number of belts consisting of several cords, which are fitted on top of the radial plies. The cords within these belts form a crown angle of 20°. The belts are provided to brace the tread and prevent buckling around the periphery of the tire. The radial-ply tire provides, relative to the cross ply, uniform ground pressure over the contact area. The length of the contact patch relative to an equivalent cross-ply tire is longer, and thus the flotation characteristics of the radial tire are better. The radial tire is also likely to develop higher levels of drawbar pull than the equivalent cross-ply tire, particularly on light, sandy loam soils. The cross ply may show some improvement in performance relative to the radial ply, in terms of pull-slip characteristics, on wet, heavy clay soils (Anslow and Warrilow, 1970). The belted bias is designed as a combination of the radial- and cross-ply tire designs, although the principal plies are arranged in cross-ply configuration, set at a low crown angle.

25.2. TIRE SELECTION

A rational approach to tire selection based on coefficient of traction determined from mobility number was proposed by Gee-Clough (1980). This approach is based on the empirical models of Wismer and Luth (Chapter 4) and of Gee-Clough et al. (1982). The dimensionless term mobility number (M) is used in the empirical relationships. It is defined as

\[
M = \frac{Cbd}{W} \left( \frac{R}{h} \right)^{1/2} \left( \frac{1}{1 + \frac{h}{2R}} \right) \]

\[\text{------------------------} (4.3)\]
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where $C$ is the soil cone index value, $b$ the tire width, $d$ the tire diameter, $h$ the tire-section height, $\delta$ the tire deflection under load, and $W$ is the dynamic load on the tire.

The tire deflection $\delta$ is normally measured statically on a hard surface, and a typical value of $\delta / h$ is 0.2 at the manufacturer's recommended load and inflation pressure. Gee-Clough took CI readings over several years and established values of 200, 700, and 1500 kPa to represent bad, average, and good field conditions, respectively. In order to accommodate the effects of various operating parameters on the prediction of tractive performance, correction factors for $(C_T)_{\text{max}}$, $K$, and $C_\tau$ were provided by Gee-Clough. These are reproduced in Table 4.1.

The correction factors are meant to be applied to a discrete situation and should not be combined. It is interesting to note that no correction factor is included for the tire aspect ratio. Change in aspect ratio will affect tractive performance; however, Gee-Clough noted that the range of aspect ratios commercially available is small and found no significant difference between a tire with an aspect ratio of 0.69 (advertised as a low aspect ratio) and a tire with an aspect ratio of 0.75.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Traction conditions</th>
<th>$(C_T)_{\text{max}}$</th>
<th>$K$</th>
<th>$C_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial ply compared to cross ply</td>
<td>Bad</td>
<td>0.95</td>
<td>1.38</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.95</td>
<td>1.38</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>0.95</td>
<td>1.38</td>
<td>1.00</td>
</tr>
<tr>
<td>High-lugged tires (i.e., 75 vs. 35 mm)</td>
<td>Bad</td>
<td>1.10</td>
<td>0.92</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1.10</td>
<td>0.92</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>1.10</td>
<td>0.92</td>
<td>1.32</td>
</tr>
<tr>
<td>Forward-speed increase (3.2 to 6.4 km/hr)</td>
<td>Bad</td>
<td>1.01</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1.01</td>
<td>1.00</td>
<td>0.98</td>
</tr>
</tbody>
</table>
In the following example, traction parameters are calculated for an axle carrying 14 kN, or 7 kN per tire. The traction parameters can be calculated as follows:

For a 9.5 x 24 agricultural drive tire with a load of 7 kN applied, the pertinent dimensions are,

- tire breadth, $b = 0.24\text{m}$
- tire diameter, $d = 1.04\text{m}$
- tire aspect ratio, $\lambda = 0.2$

For bad conditions, the cone index $C = 200\text{kPa}$.

The mobility number $M$ is given from Eq. (5-25) as

$$M = \frac{200 \times 0.24 \times 1.04}{7} \sqrt{0.2 \left( \frac{1}{1 + 0.24/2.08} \right)} = 2.85$$

The maximum coefficient of traction is given by

$$\left( C_T \right)_{\text{max}} = 0.796 - \frac{0.92}{M} = 0.796 - \frac{0.92}{2.85} = 0.47$$

The coefficient of rolling resistance is

$$C_{RR} = 0.049 + \frac{0.287}{M} = 0.049 + \frac{0.287}{2.85} = 0.15$$

The slope of the coefficient of traction versus slip curve at the origin is

$$K(C_T)_{\text{max}} = 4.838 + 0.061M = 4.838 + 0.061(2.85) = 5.01$$

The slip at maximum efficiency was shown by Gee-Clough (1980) to vary little with mobility number $M$ over the range of mobility numbers encountered for agricultural soils. It was therefore suggested that a slip value of 0.10 would represent a useful average figure for slip at maximum efficiency.

The tractive efficiency $\eta$ is
where \( C_T \) is the coefficient of traction and \( i \) the slip.

The coefficient of traction \( C_T \) is given by

\[
C_T = (C_T)_{\text{max}} (1 - e^{-ki})
\]

Where, \( K = K(C_T)_{\text{max}} / (C_T)_{\text{max}} \)

For the example given, \( K = 5.01 / 0.47 = 10.66 \). Thus,

\[
C_T = 0.47 (1 - e^{-10.66 \times 0.1}) = 0.308 , \quad \eta_{\text{max}} = \frac{0.308(1 - 0.1)}{0.308 + 0.15} = 0.61
\]

The predicted tractive performance parameters for bad, average, and good traction conditions are given in Table 4.2.

<table>
<thead>
<tr>
<th>Tractive condition</th>
<th>((C_T)_{\text{max}})</th>
<th>(K(C_T)_{\text{max}})</th>
<th>(C_{RR})</th>
<th>(h_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad</td>
<td>0.47</td>
<td>5.01</td>
<td>0.15</td>
<td>0.61</td>
</tr>
<tr>
<td>Average</td>
<td>0.70</td>
<td>5.43</td>
<td>0.08</td>
<td>0.74</td>
</tr>
<tr>
<td>Good</td>
<td>0.75</td>
<td>6.10</td>
<td>0.06</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Tire selection for drive tires can be made on the basis of tires that meet the load specifications and provide the highest tractive efficiency under the ground conditions expected. This necessitates repeating the previous calculations for each tire option. If the tires are towed, then the selection is on the basis of that tire size providing the minimum coefficient of rolling resistance \( C_{RR} \). Tire options available will, however, be limited by the range of tire sizes commercially available, and by design limitations imposed by the machine size, etc. Single and dual tires can be compared by calculating the mobility numbers for each situation (Gee-Clough, 1980). If the duals are assumed to act independently of each other, to have width \( b \), and to carry a load \( W/2 \), with diameter \( d \), the mobility number is given by

\[
M_1 = \frac{Cbd}{W} \left( \frac{\delta}{h} \right)^{1/2} \left( \frac{1}{1 + b/2d} \right)
\]

For the same tire width, the single tire will have a width \( 2b \), diameter \( d \), and carry load \( W \). The mobility number is given by
A comparison gives

\[
\frac{M_1}{M_2} = \frac{1 + (b/d)}{1 + (b/2d)} > 1
\]

The ratio is greater than 1, and thus duals will perform better than singles of the same overall dimension.

This approach can also be used to compare large single tires with smaller duals.

For duals of diameter \(d_1\), width \(b_1\), and load \(W/2\), the mobility number is

\[
M_1 = \frac{Cb_1 d_1}{W/2} \left( \frac{\delta}{h} \right)^{1/2} \left( \frac{1}{1 + (b_1/2d_1)} \right)
\]

For single tires of diameter \(d_2\), width \(b_2\), and carrying load \(W\), the mobility number is

\[
M_2 = \frac{Cb_2 d_2}{W} \left( \frac{\delta}{h} \right)^{1/2} \left( \frac{1}{1 + (b_2/2d_2)} \right)
\]

A comparison gives

\[
\frac{M_1}{M_2} = \frac{2b_1 d_1}{b_2 d_2} \left[ \frac{1 + (b_2/2d_2)}{1 + (b_1/2d_1)} \right]
\]

then \(M_1/M_2\) will be greater than 1. In this case, the duals will have better tractive performance than the single tire.
LESSON 26. BALLASTING

26.1 BALLASTING

The tractive performance of a tractor can be improved, particularly on sandy loam soils, by ballasting, which adds weight to the drive wheels. It is important, however, that the tire manufacturer's recommended load, at normal inflation pressures, not be exceeded. The addition of ballast to a tractor is usually accomplished by adding wheel weights, adding front weights, and filling the tires with water. Adding too much ballast will result in excessive power loss due to increased rolling resistance, but insufficient ballast will cause power loss because of the increased wheelslip. The driving weight required over each traction tire can be determined from the traction prediction equations of Wismer and Luth (Chapter 4). It is important to match the tractor power, weight, speed, and draft force, For a two-wheel drive tractor, Gee-Clough et al. (1982) cite Reece (1970) as given the relationship

\[
\frac{W^*}{P} = \frac{1.17}{V} \quad \text{(5.1)}
\]

and for four-wheel drive tractors

\[
\frac{W^*}{P} = \frac{0.82}{V} \quad \text{(5.2)}
\]

where \(W^*\) is the total tractor weight in kiloNewtons (kN), \(P\) the engine power (kW), and \(V\) the forward speed (m sec\(^{-1}\)). Dwyer (1978) gave a relationship of the form

\[
\frac{W}{P^*} = \frac{1.79}{V} \quad \text{(5.3)}
\]

where \(W\) is the dynamic weight on the drive wheels (kN) and \(P^*\) the total axle power (kW). This expression was obtained using average values from field tests. The equation is intended to give the weight on the drive tires per unit of axle power to ensure operation at maximum efficiency. Dwyer suggested operating at a slip of 0.1 (10%) and a coefficient of traction of 0.4.

If PTO power is to be used instead of axle power, then the losses between the axle and the PTO must be taken into account. Since PTO power levels are readily available from test reports, there is some practice value in relating dynamic weight to PTO power output. For a tractive efficiency of 0.7 and a coefficient of traction of 0.4, the PTO power, weight on the driving wheels, and the vehicle speed can be related as follows (Bloome et al, 1983):

\[
0.7(\text{PTO power}) = 0.4(\text{weight on drive wheels})(\text{vehicle speed}) \quad \text{(5.4)}
\]

This gives the relationship \(WV = 1.79 P^*\) of Eq. (5.3). Other values for coefficient of traction and tractive efficiency can be used to give different-relationships. If a 4% loss is assumed between the PTO power and the axle power, then the relationship given in Eq. (5r5) can be written as

\[
k \text{m hr}^{-1} \text{kN (PTO kW)}^{-1} = 6.20 \quad \text{(5.5)}
\]

Bloome et al. (1983) suggest that this equation is applicable to determining the optimum ballast for power-limiting conditions. The under-ballasted tractor is traction limited, and at a wheelslip of 20% and good soil conditions, a coefficient of traction of 0.5 is typical.
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Assuming a tractive efficiency of 0.7 gives an expression

\[ \frac{W}{P^*} = \frac{1.4}{V} \]  
\[ \text{(5.6)} \]

or

\[ \text{km hr}^{-1} \text{kN (PTO kW)}^{-1} = 4.84 \]  
\[ \text{(5.7)} \]

Bloome et al. (1983) discussed ballasting recommendations for two-and four-wheel drive tractors. Dwyer (1978) and a tractor manufacturer recommend the same mass-to-power relationships for two- and four-wheel drive tractors operating at the same speed. There is also a case for having a slightly greater mass-to-power ratio for two-wheel drive tractors, since some axle force must be maintained on the unpowered front wheels to provide steering control. Alternatively, it can also be argued that four-wheel drive tractors can use slightly greater mass-to-power ratios since there is no rolling resistance loss associated with unpowered wheels. Bloome et al. (1983) conclude that ballasting recommendations should be the same for two- and four-wheel drive tractors operating at the same speed. The ballast recommendations based on Dwyer (1978) are for utilization of full engine power. Tractors operating at less than full rated power will require some reduction in tractor ballast. This is perhaps best achieved for these part-load conditions by assuming appropriate values for tractive efficiency, coefficient of traction, and forward speed, and subsequently modifying Eqs. (5.5) and (5.7) in accordance with Eq. (5.4).

Gee-Clough et al. (1982) developed ballast recommendations based on the traction prediction equations developed by Gee-Clough et al. (1978). These equations were discussed in Chapter 4 and predict coefficient of traction and coefficient of rolling resistance, based on mobility number M. Using these equations, Gee-Clough et al. (1982) developed an approach for estimating the theoretical loss in tractive efficiency resulting from having drive tire loads greater or smaller than the optimum value.

The power transmitted by the drive wheels, \( P_1 \) (kN), is given by

\[ P_1 = WC_T V \]  
\[ \text{(5.8)} \]

where \( W \) is the dynamic load on the drive wheels (kN), \( C_T \) the coefficient of traction, and \( V \) the forward speed (m sec\(^{-1}\)). The maximum power that the wheels are able to transmit, \( P_2 \) (kW), at any value of slip is given by

\[ P_2 = \eta P^* \]  
\[ \text{(5.9)} \]

where \( \eta \) is the tractive efficiency and \( P^* \) the total axle power (kW).

If \( P_1 = P_2 \), then the power transmitted will be equal to the maximum possible at that value of slip, and

\[ WC_T V = \eta P^* \]  
\[ \text{(5.10)} \]

or

\[ \frac{W}{P^*} = \frac{\eta}{C_T V} \]  
\[ \text{(5.11)} \]

In order for operation at maximum power to be achieved, the slip at which \( P_1 = P_2 \) has to be the slip at maximum efficiency. Gee-Clough et al. (1982) showed that for the range of mobility numbers normally
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encountered for soil conditions (8-30), the slip at which maximum efficiency occurs is 10%. If this represents the optimum condition, then

\[
\frac{W}{P_{opt}^*} = \left(\frac{\eta}{C_T^{opt}}\right) \frac{l}{V}
\]

(5.12)

combining Eqs. (5.11) and (5.12) gives

\[
\frac{(W / P^*)}{(W / P_{opt}^*)^{opt}} = \left(\frac{C_T}{C_T^{opt}}\right) \frac{\eta}{\eta_{opt}^{opt}}
\]

(5.13)

For preset values of wheelslip and mobility number, values of \(W/P^*, C_T,\) and \(\eta\) can be found. These can be compared to the optimum values and a curve of \(W/W_{opt}\) against \(\eta / \eta_{opt}\) drawn. Gee-Clough et al. (1982) produced the diagram of this relationship, which is reproduced in Fig. 5.1. It is apparent that there is a desirable range for \(W/W_{opt}\) but that it can be allowed to deviate from \(W/W_{opt} = 1\), particularly at the increased values for mobility number.

If the power actually transmitted by the wheels, \(P_1\), is always less than the maximum power able to be transmitted, \(P_2\), at all values of slip, then the tractor will never be power limited. The conditions under which this would occur are calculated from Eq. (5.8) as

\[
P_1 = WC_T V = WC_T V_0 (1 - i)
\]

(5.14)

where \(V_0\) is the theoretical (no-load) forward speed. From Eq. (5.9),

\[
P_2 = \eta P^* = \frac{C_T (1 - i)}{C_T + C_{RR}} P^*
\]

(5.15)

and

\[
\frac{P_1}{P_2} = \frac{W V_0}{P^*} \left(\frac{C_T + C_{RR}}{C_T^{opt} + C_{RR}}\right)
\]

(5.16)
The expression $\frac{W}{P^*}$ is a dimensionless number, which is termed the ballast number by Gee-Clough et al. (1982). The tractor drive tires will never be able to deliver the power available if the ballast number is smaller than that required to give $P_1 = P_2$. This is the power-limited condition. Gee-Clough et al. (1982) showed that if the maximum slip is assumed to be 20%, then the minimum value for the ballast number to ensure the tires deliver the available power is 1.58. Thus

$$\frac{W}{P^*} = \frac{1.58}{V} (1 - 0.2) = \frac{1.26}{V}$$

This can be rewritten as

$$\text{km/hr} \cdot \text{kN (PTO kW)}^{-1} = 4.36$$

Equation (5.18) provides for a ballast level 70% of that given for the optimum ballast in Eq. (5.5). Therefore, the minimum ballast recommendation should be at 70% of that for the optimum level. Experiments by Gee-Clough et al. showed no significant reduction in power output up to a maximum ballasting of 70% of the optimum. Thus an acceptable recommendation for the ballast is that the dynamic tire load should be 70-140% of the optimum value to avoid major losses in the output power at the drawbar. The maximum ballast can be written as

$$\text{km/hr} \cdot \text{kN (PTO kW)}^{-1} = 8.68$$

To summarize:

1. The coefficient of traction at maximum efficiency can be assumed to be 0.38-0.4 for most soil conditions encountered in agriculture.

2. This level of coefficient of traction will give maximum efficiency at a slip of 10%.

Fig. 5.1. Ratio $\eta / \eta_{opt}$ Vs. $W/W_{opt}$ for mobility number $M = 3$ and $M = 30$ [from Gee-Clough et al. (1982)].
3. The load on the drive tires should be based on axle power and working speed in accordance with Eq. (5.5). That is, \( \text{km hr}^{-1} \text{kN (PTO kW)}^{-1} = 6.20 \)

The load on the tires is the dynamic tire load. It includes the static load and the weight transfer effect.

4. The minimum ballast for the traction-limited condition is given by a dynamic load that is 70% of the optimum. This is given by Eq. (5-20) as \( \text{km hr}^{-1} \text{kN (PTO kW)}^{-1} = 4.36 \).

5. The maximum ballast for the power-limited condition is given by Eq. (5.19) as \( \text{km hr}^{-1} \text{kN (PTO kW)}^{-1} = 8.68 \).

A weight transfer analysis specifically of four-wheel drive tractors was presented by Peters (1983). The equations developed were used to express the vehicle performance in terms of the rear axle. Figure 5.2 shows the free-body diagram used by Peters in analyzing the forces on a four-wheel drive tractor.

Summing moments about the rear axle gives

\[
R_i = \frac{X_1}{X_2} \left( \frac{Q_1 - (X_4 P_0)}{X_3} \right) \]  
(5.20)

where \( R_i \) is the front dynamic load, \( W \) the vehicle weight, \( Q_3 \) the total axle torque \( (= Q_1 + Q_2) \), and \( P_0 \) the resultant drawbar pull.

The front static weight is given by the term \( W(X_1/X_2) \) and the load transfer component is given by \( Q_3 - (X_4 P_0)/X_2 \).

The ballast requirement can be determined from the coefficient of traction. Equations for coefficient of traction (which is also referred to as the "dynamic ratio" and "pull ratio") were given in Chapter 4. The general expression for coefficient of traction is (Leviticus and Reyes, 1983)

Fig. 5.2. Forces acting on a four-wheel drive tractor [from Peters (1983)].
Mechanics of Tillage and Traction

\[
\frac{P}{W} = f\left(\frac{W}{bd}, t\right)
\]

And

\[
P/W = (C_T)_{\text{max}}(1-e^{-k(bd/w)i}) \tag{5.21}
\]

where \(P\) is the drawbar pull, \(W\) the dynamic weight on the tire, \(b\) the tire section width, \(d\) the tire diameter, \(i\) the slip, \((C_T)_{\text{max}}\) the constant equivalent to the maximum coefficient of traction, and \(K\) a constant related to the tire resilience (kN/m²). For a tire moving on soil, the \(K\) factor is taken to be equivalent to the cone index.

Leviticus and Reyes (1983) developed an expression termed the "tractive quotient" (TQ) from Eq. (5.21):

\[
TQ = e^{-k(bd/w)i} \tag{5.22}
\]

This expression was used to evaluate the tractive response for different tire-loading factors. Tire loading is represented by the expression \(bd/W\). Values for the constants \(A\) and \(K\) were obtained from the analysis of Nebraska tractor tests. The results were therefore only applicable to performance on concrete. As the tire-loading factor was increased, the coefficient of traction was found to increase at the same value of slip. However, the rate of increase of coefficient of traction was reduced at higher tire-loading factors.

*****😊*****
LEsson 27. Soil compaction

28.1. Soil compaction

The compaction of soil can be defined as an increase in its dry density, and the closer packing of solid particles or reduction in porosity. Compaction can result from natural causes, including rainfall impact, soaking, internal water tension and the like. Artificial compaction occurs under the downward forces of machines which are usually of short duration in the case of moving vehicles. The mechanical analysis of compaction under wheels and tracks is not simple, owing to the non-uniform nature of stress distribution, both normal and shear, in the ground. Figure 1.1, for example, shows the patterns of porosity under a rigid plunger in a Yolo sandy loam soil at 14.7 percent moisture content by weight. The greatest change in porosity is not at the plunger surface, where it is expected that the pressure is highest, but rather at the apex of a 45° isosceles triangle under the plunger face, where there is a concentration of normal stress plus shear.

A similar soil density change was observed by Gill and Reaves (1956) under smooth tires, and by Raghavan et al. (1976) in field tests using a tractor with lugged tires trailing a smooth tired sprayer. These latter results are shown in figure 1.2 as a distribution of change in soil dry density on a cross section under the tires, for different numbers of repeated passes of the machines in the same track.

It is evident in figure 1.2 that the number of times that a load is repeated affects not only the magnitude of change in soil density, but also the volume of soil which is affected. After 10 and 15 passes of the tractor and sprayer, the volume of compacted soil was seen to be progressively deeper and wider than after one or five passes. Other field tests on a clay soil at 38% surface moisture content by weight also demonstrated the increase in compacted density with higher contact pressures and number of repeated loadings, in a pattern very similar to that in figure 1.2. Figure 1.3 shows the maximum increase in dry density measured under two different tires at various average contact pressure, plotted against the number of repeated passes on a logarithmic scale. It appears that the number of repeated loadings has a similar role to the increase in contact pressure of a tire, and Raghavan et al. (1977a) suggested the following equation to describe maximum density changes, for moisture contents below the “optimum” moisture content for soil compaction.
Mechanics of Tillage and Traction

\[ \gamma_d = A + B \log(Np) + C \log(W\%) \]  
(1.1)

Where,

- \( \gamma_d \) = soil dry density (mass of solids per unit soil volume)
- A, B, C = soil constants
- N = number of repeated passes of a TIRE
- W = soil moisture content by weight (%), below the optimum.

Equation 1.1 includes the influence of soil moisture content on the degree of soil compaction for a given applied load. In fact, the humidity of the soil is a very important factor in the change of density under surface pressures.
Equation 1.1 includes the influence of soil moisture content on the degree of soil compaction for a given applied load. In fact, the humidity of the soil is a very important factor in the change of density under surface pressures.

Figure 1.4 shows the final levels of dry density in a sandy loam soil, compared to the uncompacted state, after applying various loadings by tractors of different masses and numbers of repeated passes on the surface. The standard Proctor compaction test results for this soil (Lambe, 1951) are also given in figure 1.4.

It is evident from figure 1.4 that the moisture content of this type of soil is an important factor controlling the degree to which compaction will occur under a particular load. The 15 repeated passes of a tractor with 41 kPa contact pressure, for instance, increased the original soil density by an amount about double that of a single pass at the same loading pressure. However, all of the traffic levels at the “optimum” soil moisture content for compaction, which was close to 15% by weight in this soil, produced between four and five times as much increase in soil density as they would in a dry soil state, below 5% moisture content.

*****😊*****
LESSON 28. Mechanical and hydraulic properties of compacted soil

28.1. Mechanical properties

It has been known for many years that the strength of a soil, and other mechanical properties change as the soil becomes more compact. Soil cohesion, for instance, generally rises logarithmically with soil density, while angle of internal friction tends to increase in a linear fashion with density. Table 8.1 shows some of the changes in soil mechanical properties which are affected when a soil is compacted.

The increase in soil strength following compaction is very desirable and even necessary in the construction of building foundations, roads and dams. However, it is not of benefit when soil must be subsequently excavated or tilled. Considerably more energy must be expended to cut a soil after it has been compacted, and the resultant structure of the remolded soil will most likely be different from an uncompacted soil which is excavated or worked. The moisture content of a soil affects its strength as well, generally by decreasing the cohesion and friction angle beyond the lower plastic limit. Nevertheless, at a fixed moisture content, a soil will have a higher strength at larger dry densities, which reflects the closer packing of solid particles. Figure 8.5 shows, for example, the effects of changes in both moisture content and soil dry density on the penetration resistance of a clay loam soil. It is evident that the penetration resistance, which was measured by a cone penetrometer in this case, cannot be determined by compacted density alone, but is also dependent on moisture content. The increased soil strength at higher densities will not only increase soil cutting forces and energy required, but will also impede the growth of plant roots.

Table 2.1. Change in soil mechanical properties when soil density is increased at constant moisture content

<table>
<thead>
<tr>
<th>Property</th>
<th>Soil type</th>
<th>Density range T m⁻³</th>
<th>Corresponding property range</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Clay</td>
<td>0.84 – 1.89</td>
<td>0 – 282 kPa</td>
<td>1</td>
</tr>
<tr>
<td>Φ</td>
<td>Clay</td>
<td>0.84 – 1.89</td>
<td>0 – 37°</td>
<td>1</td>
</tr>
<tr>
<td>Φ</td>
<td>Sand</td>
<td>1.74 – 2.11</td>
<td>18 – 55°</td>
<td>1</td>
</tr>
<tr>
<td>Penetration resistance</td>
<td>Clay</td>
<td>0.84 – 1.89</td>
<td>0 – 8250 kPa</td>
<td>1</td>
</tr>
<tr>
<td>Φ</td>
<td>Sand</td>
<td>1.60 – 1.72</td>
<td>27 – 32°</td>
<td>2</td>
</tr>
</tbody>
</table>
Mechanics of Tillage and Traction

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Soil Type</th>
<th>Range</th>
<th>Range</th>
<th>Source(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undrained C Clay</td>
<td></td>
<td>1.02 – 1.25</td>
<td>5 – 40 kPa</td>
<td>3</td>
</tr>
<tr>
<td>Tensile strength Loam</td>
<td></td>
<td>1.45 – 1.70</td>
<td>8 – 65 kPa</td>
<td>4</td>
</tr>
<tr>
<td>Stiffness $k_\phi$ Yolo loam</td>
<td></td>
<td>1.26 – 1.55</td>
<td>982 – 5200 kPa</td>
<td>5</td>
</tr>
<tr>
<td>Stiffness $k_\phi$ Sandy loam</td>
<td></td>
<td>1.48 – 1.73</td>
<td>2470-4800 kPa</td>
<td>5</td>
</tr>
<tr>
<td>Pulvarization energy (2.5 cm mean weight dia) Loam</td>
<td></td>
<td>1.10 – 1.56</td>
<td>22 – 173 J/kgx10^{-2}</td>
<td>6</td>
</tr>
</tbody>
</table>

1. Proctor (1948)
2. Taylor (1948)
3. Graecon (1960)
4. Vomocil et al. (1961)
5. Chancellor and Schmidt (1962)
6. Bateman et al. (1965)

Fig 2.1. The penetration resistance (cone index pressure) on a 12.8 mm cone (ASAE, 1984) in a clay loam soil at different dry densities and moisture contents by weight (Taylor et al. 1981)

28.2 Hydraulic properties

In a review of the effects of soil compaction on the content and transmission of water in soils, Warkentine (1971) noted that compaction alters the water content and movement in soils by modifying
the void size distribution. Large macropores in the soil fabric are the first to be reduced in volume, as shown in figure 8.6. This tends firstly to reduce the amount of water which is retained at low water suction pressures in the macropores, and secondly to reduce the saturated hydraulic conductivity of the soil.

![Fig 2.2. Schematic representation of large macropores in soil fabric before and after compaction](image)

The reduction in saturated hydraulic conductivity of soils is generally logarithmic with changes in soil dry density, or with void ratio. Some of the changes of observed alterations in clay and loam soils are given in table 2.2.

**Table 2.2. Observed changes in saturated hydraulic conductivity at different soil dry densities**

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Range of dry density, t m$^{-3}$</th>
<th>Range of saturated hydraulic conductivity, $10^{-6}$ cm s$^{-1}$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beach sand</td>
<td>1.40 – 1.66</td>
<td>55000 - 20000-</td>
<td>1</td>
</tr>
<tr>
<td>Dam filter sand</td>
<td>1.66 – 1.83</td>
<td>13000 – 7000</td>
<td>2</td>
</tr>
<tr>
<td>Dike sand</td>
<td>1.24 – 1.61</td>
<td>1800 – 150</td>
<td>2</td>
</tr>
<tr>
<td>Ste-Rosalie clay</td>
<td>1.15 – 1.55</td>
<td>2700 – 100</td>
<td>3</td>
</tr>
<tr>
<td>Yolo silt loam</td>
<td>1.20 – 1.50</td>
<td>3000 – 30</td>
<td>4</td>
</tr>
<tr>
<td>North Carolina silt</td>
<td>1.43 – 1.66</td>
<td>60 – 0.6</td>
<td>1</td>
</tr>
<tr>
<td>Boston silt</td>
<td>1.27 – 1.51</td>
<td>1 – 0.002</td>
<td>2</td>
</tr>
</tbody>
</table>
As table 2.2 shows, the hydraulic conductivity of a saturated soil can change by one or two orders of magnitude when compaction occurs. The reduction in size of macropores, shown in figure 2.2 has other consequences for the water status in soils as well. The amount of water which can be stored and easily available for plant use will also be reduced, as well the quantity of air in the soil (of which about 10% of the total volume is needed for healthy plant roots). The drainage of a soil will also be slowed, which leads more readily to high degrees of saturation in rainy periods of the year, and an insufficient quantity of air for crop roots.

On the other hand, there is the possibility that a soil is not compacted enough. This arises in all soil types when the dry density is low, the volume of macropores is large and drainage may be too rapid. In such case, especially during periods of low precipitation, there may be insufficient water stored in the arable soil profile, and an excess of air. Plant roots then need to seek water stored in the micropores under high suction, and eventually dry the soil to the “wilting point” at which water content they can no longer extract moisture from soil fabric.

From the view point of soil hydraulic properties, there is actually no unique optimum dry density of a soil. The best structure is dependent on the type of crops growing on a field, their rooting depth and water extraction capability and the quantity of precipitation input, which varies from place to place, and even from year to year at a single location.
LESSON 29. SOIL PHYSICAL PROPERTIES AND PLANT GROWTH

29.1. Soil physical properties and plant growth

The shear strength, penetration resistance and hydraulic properties of soils change according to the level of compaction. Both these mechanical and hydraulic properties have an effect on the rate at which plant roots can grow to depth in a soil, and the flow and availability of water and nutrients for plant use. This fact has been quite obvious in numerous comparative observations of root and crop growth in different soils.

Taylor et al. (1966), for example, measured the number of tap roots of the cotton plant which penetrated compacted layers of different soils, and characterized the degrees of compaction by means of measurements with a cone penetrometer. As their results in figure 3.1 indicate, the number of roots penetrating the soil was reduced drastically as the penetration resistance approached 2 MPa pressure. In fact, at soils compacted to more than 2 MPa resistance, virtually no roots at all were able to grow.

Similarly, Raghavan et al. (1979) dug trenches in an experimental clay field of silage corn, in which plots had been treated to varying levels of compaction by machinery traffic. Washed root samples at different depths for each level of soil compaction were weighed, and the results are shown in figure 3.2 as cross sectional maps of root distributions at harvest time for the various treatments.
Fig 3.2. Root density distributions in a clay field of silage corn wherein plots were subjected to different levels of compaction by machinery traffic, at an average compact pressure of 61.7 kPa (Raghavan et al. 1979)

The heaviest compaction treatment (15 tractor passes) evidently decreases the maximum depth of rooting by one half, and the depth of dense roots to about one third of that in uncompacted soil. This effect, together with soil water status alterations had the net result of considerably reducing the growth and yield of the silage corn compared to uncompacted plots in the same field, (figure 3.3).
Fig 3.3. Dry matter yield of silage corn as a function of soil dry density between 0 and 20 cm depth in a clay field of silage corn

The results of figure 3.3 were obtained in successive plantings of the same hybrid of silage corn on a field of clay in the years 1976, 1977 and 1980. The various densities of top soil were obtained by applying different levels of machinery traffic in plots following an initial rotary cultivation of the field in the spring of each year to a depth approximately 25 cm. Between the years 1977 and 1980, the yields in plots of different density were close together. In 1976, the yields were higher at lower density, around 16 t ha$^{-1}$ at 1 t m$^{-3}$. The curves in figure 3.3 fitted to the data points illustrate two important aspects of the effects of soil density upon crop growth, namely phenomenon of optimum density, and the role of local seasonal precipitation.

The occurrence of an optimum density for soil was observed by Vomocil (1955) and reported by Rosenberg (1964). Vomicil noted the yields of field corn, sweet corn and potatoes on some New Jersey soils, at different degrees of compaction, and concluded that a soil which is either looser or more dense than a particular density value will incur yield losses compared to that optimum structure. He proposed an equation to describe this phenomenon with a parabolic shape, such that the loss in crop yield increases as the square of the difference in soil dry density from the optimum, as shown in equation 3.1.

$$Y^* - Y = C(Y_{\text{dry}}^* - Y_{\text{dry}})^2$$

(3.1)

Where,

$$Y^* = \text{the maximum obtainable crop yield}$$
Y = the actual crop yield

C = a constant

\( Y_{\text{dry}} \) = the actual soil dry density (averaged from 10-40 cm depth)

\( \gamma_{\text{dry}}^* \) = the optimum soil dry density for maximum yield

Vomocil (1955) suggested that the constant C, which can be referred to as a factor of sensitivity to soil compaction, depends upon both the type and variety of crop, and the weather. The results of figure 3.3 confirm the concept of a parabolic change in crop yields. However, there appears to be a distinct difference in yield behaviour between the 1976 and the other two years’ seasons. The rainfall pattern was also markedly different in these periods, and may well provide a sufficient explanation for the different behaviour. In 1976, one of the wettest summers on record in the Montreal area, 330 mm of rain fell in the combined months of June, July and August. In the years 1977 and 1980, 215 and 220 mm, respectively, were experienced in the same period. These latter two amounts of precipitation are essentially the same, as are the silage corn yields at each soil density in figure 3.3 for those years.

With about 50% more rainfall in 1976, not only has the sensitivity factor C of equation 8.2 diminished from 282 to 90 t ha\(^{-1}\) per t m\(^{-3}\), but the apparent optimum soil dry density has been reduced as well from about 1.13 to 0.99 t m\(^{-3}\), and the maximum possible crop yield, all other factors being equal, has increased from 12.3 to 16 t ha\(^{-1}\). It would appear then that it is not only the compaction sensitivity constant C, which is dependent on annual weather patterns, but also the optimum soil density, and the maximum obtainable crop yield.

Such behaviour is reasonable in view of the preceding comments on soil hydraulic properties as a function of compaction. A loose soil with large macropores, such as the clay of figure 8.9 at a dry density of 1 t m\(^{-3}\) and porosity 62%, might normally be expected to drain too quickly, and to retain an insufficient quantity of moisture during dry periods of the growing season for optimum crop growth. However, during a year such as 1976, when the summer rainfall exceeded the normal average by some 50%, there were no extended dry periods at any time in the growing season. Therefore, the loose soil fabric, with its reduced impedance to root growth and increased conductivity for moisture and nutrients, allowed the soil-plant system to take advantage of the plentiful rainfall and to produce higher than normal yields.

The years 1977 and 1980 represented average weather conditions from June to September, and were thus more typical of what is to be expected in most years. In such a case, the soil density of 1 t m\(^{-3}\) was too low for water retention in periods between rainfalls, which extended up to ten days in those years. The available water in the soil fabric was used to an extent that there was considerable water stress in the plants at some times, and over the growing season the total crop yields were reduced by some 30% below the optimum quantity which occurred at a somewhat higher soil density (1.13 t m\(^{-3}\)).

In all of the years of the study illustrated in figure 8.9, and in all studies reported on the effects of compaction on crop growth, an excessive amount of compaction above the optimum dry density also results in reduced crop growth. In the case of clay soil of figure 3.3, the dropping off of yields was quite rapid, with nearly half of the yield being lost for a dry density increase of only 0.13 t m\(^{-3}\) above the optimum. Other soil types such as the sandy loam of figure 3.4, appear to be less sensitive to the absolute amount of density change. The difference here is logical when one considers that there is a much larger discrepancy between the sizes of macropores and micropores in a clay soil, and only small overall density changes may be needed to close off the macropores among clay structural units. In a sandy soil, however, there is a more gradual distribution of macro and micropore volumes among soil
particles, and larger overall density increases are required in order to have an equivalent effect on soil structure as it influences the movement of roots and moisture in the growing season.

Fig 3.4. Relationship between silage corn crop yields and the dry density of the 0-20 cm layer of a sandy loam soil (Negi et al., 1981)

In a detailed study of root growth and water movement in field plots of silage corn on clay soil, Douglas and McKyes (1983) outlined the stresses which can arise in soils compacted or tilled to different structures. Figure 3.5 shows an example of the rate of water extraction from corn roots at varying depth in the soil, along with the dry density patterns in the corresponding plots of different cultural treatments. The roots in the plots tilled by a moldboard plow and a subsoiler were not active as deep as the roots in other treatments, most likely owing to a dense “pan” layer of soil occurring at the 20-25 cm depth.

Fig 3.5. Patterns of corn root extraction rate of water at depths in clay soil plots subjected to different tillage treatments, after 57 DAS, and the dry density profiles of the various plots (Douglas and McKyes (1983))

This phenomenon shows up also in figure 3.6 which contains plots of crop growth stress factors as a function of time during the growing season. The two crop stress factors shown, one related to water deficits and the other to root growth impedance, were calculated as the difference between the initial and the later rates of crop growth divided by the initial and subsequent actual water transpiration rate, and effective rooting depth respectively.
The curves in figure 3.6 indicate that both the loose “control” clay plots and the compacted untilled plots suffered more water stress for crops than the others. However, it was the moldboard plow and subsoiler treatments which resulted in greater stresses for root growth. In this case, the root impedance stresses in the moldboard plow and subsoiler treatments were more severe than water availability stresses, because as the results of figure 3.7 show, the final crop yield was lower in these treatments than in the loose control soil. The compacted untilled treatment caused the highest soil packing of all, and the combination of root and water stresses led to low corn yields. The pattern of crop yield versus soil dry density in figure 3.4 is similar to that described by the Vomocil (1955) equation 3.1 and those of figures 3.3 and 3.4.

Fig 3.7. Dry matter harvest yield of silage corn on plots of clay soil subjected to different treatments of compaction and tillage as a function of soil dry density (Douglas and McKyes (1983))
LESSON 30. MEASURES FOR OPTIMIZING CROP GROWTH BY AVOIDING EXCESSIVE SOIL COMPACTION

30.1. MEASURES FOR OPTIMIZING CROP GROWTH

The following measures for optimizing crop growth by avoiding excessive soil compaction can be derived from the above results:

1. Avoid high machinery contact pressures, especially during repeated passes on fields. For a cultivation program which requires between five and ten passes of machines on the field per year, it is recommended that the tire contact pressure of the vehicles involved be limited to less than 70 kPa.

2. Avoid if possible, travelling on fields with machines when the top soil is moist, close to the “optimum” moisture content for compaction. Densification of soil can be up to five times as severe at the optimum water content under a given compacting pressure as when the soil is quite dry.

3. Avoid excessive slipping of tractor tires during field operations, which could double soil density changes under the same weight. Undue rates of wheel slip also lead to premature wear and costly replacement of tires. A maximum slip rate of 16% is recommended.

4. Attempt to manage cultural programs such that a healthy system of strong roots, and sufficient organic matter remain in the top soil. Compaction studies on a vigorous cereal stubble have shown that an extensive root system near the soil surface can reduce compaction damage under machinery loads by about two thirds compared to bare soil with low organic matter content (Chasse et al., 1975).

30.2. TILLAGE OF COMPACTED SOIL

Compaction increases the strength of a soil, and the energy which is required in order to cut or till it. Even when sufficient extra energy is expended to cut and loosen a compacted soil structure, the resulting structure will probably not be the same as that of the original uncompacted state. Studies of possible methods by which to alleviate the effects on soil structure caused by heavy construction machinery, for instance, have indicated that the cutting and lifting of a severely compressed clayey or silty soil results in a rather blocky structure. Large clods of 10 to 20 cm sizes are separated by the tillage action, but these structural units are relatively compact, hard and impervious in themselves. Furthermore, it is very difficult to refine the top soil texture subsequently, by means of conventional secondary tillage tools such as discs, cultivators and the like.

Cutting and loosening is about the only practice which can begin to improve a compacted and damaged top soil structure. With the years, the open spaces between clods will allow the passage of roots, moisture and air. And their action will eventually enter into the compacted soil blocks, to gradually open larger pores through root penetration, wetting and drying, and the overall soil tilth will experience improvement. What cannot be expected is that a single loosening tillage action will immediately restore a healthy compacted soil profile to its former uncompacted structural quality.
LESSON 31. GEOSTATISTICS / KRIGING

31.1 Introduction

Most of the naturally occurring phenomena are variable both in space and time. Considering a topographic surface or a soil property variability one can observe high variability within small distances. The variability is a result of natural processes and hence deterministic. As most of these processes are sensitive and the conditions under which they take place are not fully known, it is not possible to describe them based on physical and chemical laws completely.

It is not uncommon to use probabilistic and statistical methods for describing partly known (or sampled) natural parameters. Time series analysis is one of the earlier fields where variability has been considered and described with stochastic methods. These methods were extended and further developed to analyse spatial variability. These spatial methods form the discipline called geostatistics.

Geostatistics is a tool to help us characterize spatial variability and uncertainty resulting from imperfect characterization of that variability. In its broadest sense, geostatistics can be defined as the branch of statistical sciences that studies spatial/temporal phenomena and capitalizes on spatial relationships to model possible values of variable(s) at unobserved, unsampled locations (Caers, 2005).

31.2 Objectives of geostatistics

- To find out the weighted average of any property which varies from point to point over a given area of land for result interpretation and for carrying out simulation experiments in the field.
- To work out interpolated values of a given property over time and space in unsampled or unvisited sites between sampled estimates for the purpose of depicting contour lines on the base maps.

31.3 Utilities of geostatistics

- Determination of the exact point at which maximum variability of a given property, under a given set of conditions, may be encountered.
- Mapping of a given area for a particular property can be accomplished in the most economical way, without actually surveying the entire area on a point basis.
- The results can most suitably be extended to development of stochastic models for any given property.
- The technique can help in conservation of resources by way of simply extrapolating the results on spatial analysis in a given sampling unit(s) to similar units established or reported elsewhere.

31.4 Basics of Geostatistics

Geostatistics involves the theory of regionalized variables, which dates back to the early fifties when in South-Africa D. Krige and his colleagues started to apply statistical techniques for ore reserve
estimation. In the sixties, the French mathematician G. Matheron gave theoretical foundations to these methods.

Application of Geostatistics in its nascent stages was oriented towards the mining industry, as high costs of drillings made the analysis of the data extremely important. Books and publications on geostatistics are mostly oriented to mining problems. The phenomenal growth of computer era has lead to affordable and cheaper computational methods and hence scope of geostatistics has drastically widened. Of late, applications of geostatistics can be found in very different disciplines ranging from the geology to soil science, hydrology, meteorology, environmental sciences, agriculture and even structural engineering.

Typical questions of interest to a geostatistician are (Hengl, 2009):

- How does a variable vary in space-time?
- What controls its variation in space-time?
- Where to locate samples to describe its spatial variability?
- How many samples are needed to represent its spatial variability?
- What is a value of a variable at some new location/time?
- What is the uncertainty of the estimated values?

Geostatistics are based on the concepts of regionalized variables, random functions, and stationarity. A brief theoretical discussion of these concepts is necessary to appreciate the practical application of geostatistics.

### 31.4.1 Regionalized variable and random function

A regionalized variable is the realization of a random function. This means that for each point \( u \) in the \( d \)-dimensional space the value of the parameter we are interested in, \( z(u) \) is one realization of the random function \( Z(u) \). This interpretation of the natural parameters acknowledges the fact that it is not possible to describe them completely using deterministic methods only. In most cases it is impossible to check the assumption that the parameter is the realization of a random function as we have to deal with a single realization.

### 31.4.2 Stationarity

A random function \( Z(x) \) is said to be first-order stationary if its expected value is the same at all locations throughout the study region.

\[
E(x) = m \quad (1)
\]

Where, \( m \) is the mean of classical statistics, and

\[
E Z(x) - Z(x - h) = m \quad (2)
\]

Where, \( h \) is the vector of separation between sample locations.

Geostatistical methods are optimal when data are
Mechanics of Tillage and Traction

- normally distributed and
- stationary (mean and variance do not vary significantly in space)

Significant deviations from normality and stationarity can cause problems, so it is always best to begin by looking at a histogram or similar plot to check for normality and a posting of the data values in space to check for significant trends.

There are three scientific objectives of geostatistics (Diggle and Jr, 2007):

1. Model estimation, i.e. inference about the model parameters;
2. Prediction, i.e. inference about the unobserved values of the target variable;
3. Hypothesis testing;

Model estimation is the basic analysis step, after which one can focus on prediction and/or hypothesis testing. In most cases all three objectives are interconnected and depend on each other. The difference between hypothesis testing and prediction is that, in the case of hypothesis testing, we typically look for the most reliable statistical technique that provides both a good estimate of the model, and a sound estimate of the associated uncertainty. It is often worth investing extra time to enhance the analysis and get a reliable estimate of probability associated with some important hypothesis, especially if the result affects long-term decision making. The end result of hypothesis testing is commonly a single number (probability) or a binary decision (Accept/Reject). Spatial prediction, on the other hand, is usually computationally intensive, so that sometimes, for pragmatic reasons, naïve approaches are more frequently used to generate outputs; uncertainty associated with spatial predictions is often ignored or overlooked. In other words, in the case of hypothesis testing we are often more interested in the uncertainty associated with some decision or claim; in the case of spatial prediction we are more interested in generating maps (within some feasible time-frame) i.e. exploring spatio-temporal patterns in data.

Spatial prediction or spatial interpolation aims at predicting values of the target variable over the whole area of interest, which typically results in images or maps. In geostatistics, interpolation corresponds to cases where the location being estimated is surrounded by the sampling locations and is within the spatial auto-correlation range. Prediction outside of the practical range (prediction error exceeds the global variance) is then referred to as extrapolation. In other words, extrapolation is prediction at locations where we do not have enough statistical evidence to make significant predictions.

An important distinction between geostatistical and conventional mapping of environmental variables is that geostatistical prediction is based on application of quantitative, statistical techniques. Until recently, maps of environmental variables have been primarily been generated by using mental models (expert systems). Unlike the traditional approaches to mapping, which rely on the use of empirical knowledge, in the case of geostatistical mapping we completely rely on the actual measurements and semi-automated algorithms.

In summary, geostatistical mapping can be defined as analytical production of maps by using field observations, explanatory information, and a computer program that calculates values at locations of interest (a study area). It typically comprises:
31.4.3 Spatial Prediction Models

Spatial prediction models (algorithms) can be classified according to the amount of statistical analysis i.e. amount of expert knowledge included in the analysis:

31.4.3.1 Mechanical (Deterministic) Models

These are models where arbitrary or empirical model parameters are used. No estimate of the model error is available and usually no strict assumptions about the variability of a feature exist. The most common techniques that belong to this group are:

1. Thiessen polygons;
2. Inverse distance interpolation;
3. Regression on coordinates;
4. Natural neighbors;
5. Splines;

31.4.3.2 Linear Statistical (Probability) Models

In the case of statistical models, the model parameters are commonly estimated in an objective way, following probability theory. The predictions are accompanied with an estimate of the prediction error. A drawback is that the input data set usually needs to satisfy strict statistical assumptions. There are at least four groups of linear statistical models:

- kriging (plain geostatistics)
- environmental correlation (e.g. regression-based)
- Bayesian-based models (e.g. Bayesian Maximum Entropy)
- hybrid models (e.g. regression-kriging)

31.5 Kriging

Most geostatistical studies in natural resources management aim at estimating naturally occurring phenomena at unsampled places and mapping them. Kriging is a generic name adopted by the
geostatisticians for a family of generalized least-squares regression algorithms (Webster, 1996). There are many different kriging algorithms, and most important of them are discussed below.

(1) Punctual Kriging

Punctual Kriging is a means of local estimation in which each estimate is a weighted average of the observed values in its neighbourhood. The interpolated value of regionalized variable \( z \) at location \( x_0 \) is,

\[
Z(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i) \quad \ldots \ldots \ldots \ldots \ldots \ldots (3)
\]

When \( n \) is the number of neighbouring samples \( z(x_i) \) and \( \lambda_i \) are weights applied to each \( x(x_i) \).

(2) Block Kriging

In block kriging, a value for an area or block with its centre at \( x_0 \) is estimated rather than values at points as in punctual kriging. The kriged value of property \( z \) for any block \( v \) is a weighted average of the observed values \( x_1 \) in the neighbourhood of the block i.e.

\[
Z(v) = \sum_{i=1}^{n} \lambda_i Z(x_i) \quad \ldots \ldots \ldots \ldots \ldots \ldots (4)
\]

(3) Co-kriging

The co-regionalization of two variables \( z_1 \) and \( z_2 \) is summarized by the cross-semi variogram.

\[
\gamma_{1z}(h) = \frac{1}{2Nh} \sum_{i=1}^{n} (Z_1(x_i) - Z_1(x_i + h)(Z_2(x_i) - Z_2(x_i + h)) \quad \ldots \ldots \ldots \ldots \ldots \ldots (5)
\]

Co-kriging has been applied only to point estimation of soil properties.

(4) Universal Kriging

Universal Kriging was designed to permit kriging in the presence of trends in the sample data i.e.

\[
E(Z(X)) = m = \sum_{i=1}^{m} a_i f_i(X) \quad \ldots \ldots \ldots \ldots \ldots \ldots (6)
\]

\( E(Z(x)) \) is the expected value of the sample data, \( m(x) \) is the trend, \( f_i \) are the terms in the polynomial and are the co-efficient of the polynomial that describes the trend.
LESSON 32. GIS FOR SOIL VARIABILITY STUDIES

32.1. Introduction

Geographic Information System (GIS) is defined as an information system that is used to input, store, retrieve, manipulate, analyze and output geographically referenced data or geospatial data, in order to support decision making for planning and management of land use, natural resources, environment, transportation, urban facilities, and other administrative records.

A GIS thus consists of

(a) An extensive database of geographic information involving both positional data about land features and descriptive/non-locational data about these features at different points of time and

(b) Sets of programmes of applications, which enable the data to be input, assessed, manipulated, analysed and reported

32.2. Components of GIS

- Hardware
- Software
- Data
- People
- Methods

(i) Hardware

Hardware is the computer on which a GIS operates. Today, GIS software runs on a wide range of hardware types, from centralized computer servers to desktop computers used in stand-alone or networked configurations.

(ii) Software

GIS software provides the functions and tools needed to store, analyze, and display geographic information. Key software components are:

- Tools for the input and manipulation of geographic information
- A database management system
- Tools that support geographic query, analysis and visualization
- A graphical user interface (GUI) for easy access to tools
Data

Possibly the most important component of a GIS is the data. Geographic data and related tabular data can be collected in-house or purchased from a commercial data provider. A GIS will integrate spatial data with other data resources and can even use a DBMS, used by most organizations to organize and maintain their data, to manage spatial data.

People

GIS technology is of limited value without the people who manage the system and develop plans for applying it to real-world problems. GIS users range from technical specialists who design and maintain the system to those who use it to help them perform.

Methods

A successful GIS operates according to a well-designed plan and business rules, which are the models and operating practices unique to each organization.

32.3. Advantages of GIS

- Exploring both geographical and thematic components of data in a holistic way
- Stresses geographical aspects of a research question
- Large volumes of data
- Integration of data from widely disparate sources
- Allows a wide variety of forms of visualisation

32.4. Disadvantages of GIS

- Data are expensive
- Learning curve on GIS software can be long
- Shows spatial relationships but does not provide absolute solutions
- Origins in the Earth sciences and computer science. Solutions may not be appropriate for humanities research

32.5. Spatial variability of soil characteristics

Characterization of spatial variability of soil physical and chemical characteristics (e.g., soil texture, organic matter, salinity, water content, compaction, and nutrient content) is very important for managing agricultural practices. The precision of statements that can be made about soil properties at any location depends largely on the amount of variation within the area sampled. As heterogeneity of soils increases, the precision of statements about their properties, behavior, and land use performance decreases.

Spatial variability of soil variables is commonly a result of complex processes working at the same time and over long periods of time, rather than an effect of a single realization of a single factor. To explain
variation of soil variables has never been an easy task. Many soil variables vary not only horizontally but also with depth, not only continuously but also abruptly. Field observations are, on the other hand, usually very expensive and we are often forced to build 100% complete maps by using a sample of less than or equal to 1%.

32.6. Objectives of soil spatial analysis

Spatial analysis of soils also known as neighbourhood analysis has the following objectives.

- To find out the weighted average of a given soil property which varies from point to point over a given area of land for result interpretation and for carrying out simulation experiments in the field.
- To work out interpolated values of a given soil property over time and space in unsampled or unvisited sites between sampled estimates for the purpose of depicting contour lines on the base maps.
- To develop a rational sampling strategy for characterization of soil status to pave the way for successful implementation of field experiments.

The advent of GIS softwares has simplified the process of studying the variability with geostatistics being part of every GIS software.

32.7. Geostatistics

Geostatistics is a tool to help us to characterize spatial variability and uncertainty resulting from imperfect characterization of the variability. Geostatistics involves the theory of regionalized variables, which dates back to the early fifties and includes concepts of random function and stationarity. Geostatistical mapping can be defined as analytical production of maps by using field observations, explanatory information, and a computer program that calculates values at locations of interest. There are a number of spatial prediction models depending on the amount of statistics involved in the analysis.

Most geostatistical studies in soil variability studies aim at estimating soil properties at unsampled places and mapping them. Kriging is a generic name adopted by the geostatisticians for a family of generalized least-squares regression algorithms.

32.8. Interpolation by Kriging

Kriging is a technique of making optimal, unbiased estimates of regionalized variables at unsampled locations using the structural properties of the semivariogram and the initial set of data values. A useful feature of kriging is that an error term (estimation variance) is calculated for each estimated value providing a measure of the reliability of the interpolation. The simplest forms of kriging involve estimation of point values (punctual kriging) or areas (block kriging) and assume that the sample data are normally distributed and stationary. Various other estimation procedures are available when sample data show departures form these assumptions.

Soil properties often exhibit lognormal or complex probability distributions, in which case lognormal or disjunctive kriging is more appropriate. Directional differences in variation can also be taken into account during interpolation by using the anisotropic semivariogram model to obtain the weights in the kriging system.
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